

How to write mathematics

Research Methodology

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1 December

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It takes practice to be able to write mathematical text.

Inserting formulas in ordinary text usually causes problems.

Mathematical texts are usually very compact and require concentration and time to understand.

1. A mathematical text has to be readable. Do not use too many symbols and write complete sentences.
2. All formulas has to be a part of a sentence.
3. Never start a sentence with a formula.
4. If a formula ends a sentence, add a dot after the formula.
5. A definition should be explained the first time it is introduced.

6. Keep the reader informed.

We start with ...

Next step is technical, but important ...

We have showed that ...

7. Be clear about what you mean. Do not write:

According to a theorem in an old book is...

8. Present the result in a logical way.

9. It is important to think about how a formula is pronounced. The formula should be readable, from left to right, no matter how complicated the formula is.

Example

we have that $a < b$

since $a > b$

$$\int_a^b e^x dx$$

Pronounced

we have that a is smaller than b

since a is greater than b

the integral from a to b of e

to the power of x , dx

10. A well thought-out choice of labels can increase the readability of the document.
11. Letters that denote variables, constants, functions, . . . , should be written in italic.
12. Use the same style in the whole document.
13. Do not over use equivalence- and entailment-arrows in your text.

14. Never use relations symbols as abridgment.

Do not write: Let k be an integer > 1 .

Write: Let k be an integer larger than 1.

Or write: Let $k > 1$ be an integer.

15. Use the correct symbol for multiplication. Usually you don't need to write out the multiplication, but some times it will increase the readability of the text.

Write $\frac{2}{3} \cdot \frac{4}{5}$ instead of $\frac{2}{3}\frac{4}{5}$.

16. Avoid the symbol i as a summation index if your text also contains complex numbers.

17. Ellipsis is used with enumeration. We use ellipsis to mark that the number of objects is too many to fit in the text.
18. Ellipsis should always consist of exactly 3 dots.
19. Comma characters should be used in a finite enumeration.

Write x_1, x_2, \dots, x_n instead of x_1, x_2, \dots, x_n

Write $1 + 2 + \dots + n$ instead of $1 + 2 + \dots + n$

Write $a_1 \cdot a_2 \cdot \dots \cdot a_k$ instead of $a_1 \cdot a_2 \cdot \dots \cdot a_k$

20. Infinite enumerations should be written in a different way,

Write $(a_n)_{n=1}^{\infty}$ instead of a_1, a_2, \dots

Write $\sum_{k=1}^{\infty} \frac{1}{k}$ instead of $1 + \frac{1}{2} + \frac{1}{3} + \dots$

21. A forth dot should be written if an infinite enumeration ends a sentence.

22. Words in index shall be written in straight style.

x_{\max} instead of x_{\max} .

23. Do not over use index and exponentials. Try to avoid expression like

$$Z_{x_a^b}^{y_c^d}.$$

24. Paranthesis are very important when you write mathematics. A missplaced or left out paranthesis change the purpose of the text. The following examples does not mean the same thing

$$(a + b)/c \text{ and } a + b/c$$

$$\log x + 1 \text{ and } \log(x + 1)$$

$$\sin(x/2) \text{ and } (\sin x)/2.$$

25. Paranthesis ussually come in pairs. A left paranthesis and corresponding right paranthesis shall be written in the same size.
26. Use the correct size on the paranthesis:

$$\left(\frac{2}{x} + 1\right) \text{ instead of } \left(\frac{2}{x} + 1\right)$$

27. Formulas that are too long, too wide or too important to fit inside ordinary text should be written on a separate line.
28. Stand-alone formulas have to be a part of a complete sentence.
29. Do not end a part with a stand-alone formula.

30. A stand-alone formula shall have a number if and only if you want to refer to the formula.
31. If the number of the formula has parenthesis around it, so should the cross-reference.

32. Do not use slash if the formula is to high.

Write $\int_a^b \frac{f(x)}{x} dx$ instead of $\int_a^b f(x)/x dx$

Do not use slash if the formula has to many paranthesis

Write

$$\frac{a(1 + b) + b(6 - a)}{c(7 - b) - d(f + a)}$$

instead of

$$(a(1 + b) + b(6 - a))/(c(7 - b) - d(f + a))$$

33. Write paranthesis or brackets around matrices:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ or } \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

34. Vertical lines determines a determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}.$$

You should write $\det(A)$, not $|A|$ if you mean the determinant of a matrix A .

35. Functions that are abridgments are written in straight style. For example \sin , \lim , \arctan , \max ,
36. Sigma operators shall be written in a smaller style when they are written in ordinary text, comparing to stand-alone formulas.

37. Theorems and proofs should be easy to find in the document, for example make the text "Proof" bold. All theorems shall have a number.
38. All assumptions in a theorem/proof should be written.
39. The theorem should be written before the proof.
40. Mark the end of the proof with either the text "Q.E.D" or the symbol \square .
41. A proof should be easy to follow. It is better to write to many steps than to few.

42. Try to avoid word wrapping formulas.
43. If you have to word wrap a formula/expression try to do it in connection to an equal or plus sign ($=$, $+$).

44. Rationally expressions have a tendency to be large inside the text. Sometimes it is better to write a/b or ab^{-1} instead of $\frac{a}{b}$.
45. Use exponent when you have large expression under a square root sign:

Write $\left(\sum_{n=1}^{\infty} a_n\right)^{1/2}$ instead of $\sqrt{\sum_{n=1}^{\infty} a_n}$.

46. Avoid many stress marks on top of each other, for example $\hat{\hat{u}}$.
47. Do not forget to write out zero even if the integer part is equal to zero in a decimal number,

Write 0,1234 instead of ,1234.

3 Examples

Example:

Study the sentence:

There exists many prime numbers.

The word "many" is not a good way to express the amount of prime numbers. A better word is "infinitely".

Write:

There exists infinitely many prime numbers.

3 Examples

Example:

Study the sentence:

For all z , the real part ≥ 5 is $f(z)$ constant.

The symbol " \geq " should not be used as an abbreviation of text.

Write instead:

For all z such that $\operatorname{Re}(z) \geq 5$ is $f(z)$ constant.

Or write:

The function $f(z)$ is constant for all z whose real part is greater or equal to 5.

3 Examples

Example:

Study the sentence:

A quadratic invertible matrix.

$$\det(A^{-1}) = \frac{1}{\det(A)}.$$

The first sentence start with a formula. The second sentence is just a formula. Both sentences are incomplete. It is possible to write both sentences as one sentence.

Write:

If A is a quadratic invertible matrix, then is

$$\det(A^{-1}) = \frac{1}{\det(A)}.$$

Find all mistakes in the following exercises. Try to formulate the sentence better.

1. Let n be an integer and let $k = 2n + 1$. K is odd

2.

$$x^4 - 4x^2 = -4 = (x^2)^2 - 4x^2 + 4 = 0 = (x^2 - 2)^2 = x^2 = 2 = x = \pm\sqrt{2}$$

3. n is a prime number iff $\prod_{y=2}^{n-1} \prod_{z=1}^{n/y} (n - yz) > 0$.

4. p prime number, $p|ab \Rightarrow p|a \vee p|b$.

5. If $f(x)$ is real & odd function.

6. We have that $-i^2 = -1$.

7. If n is a positive integer, then is $\Gamma(n) = n!$

1. Keep the mathematical language on a good level.
2. An easy way to find mistakes in your text is to read the text loud for your self, including all formulas.
3. Be consistent. Do not have two different labels for the same thing.
4. Proofread at least once. Do not trust the spelling function in your computer.
5. Write a lot. The only way to learn is to practice.

Nicholas J. Higham (1993), *Handbook of Writing for the Mathematical Sciences*, Philadelphia: SIAM.

Nyqvist Robert (2006), *Matematiska skrivregler*, Växjö: Växjö universitet MSI.