Discrete Math 24/5 2019.

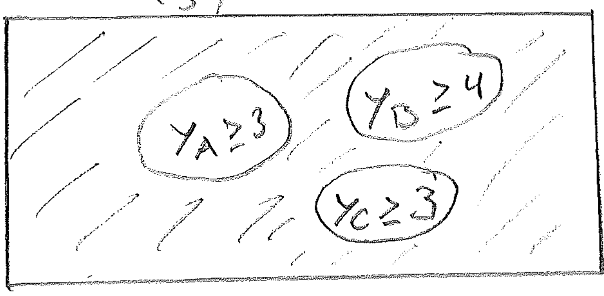
1. a) $X_A = 2 + Y_A \qquad 0 \le Y_A \le 2$

$X_B = 5 + Y_B \qquad 0 \le Y_B \le 3$

$X_C = 3 + Y_C \qquad 0 \le Y_C \le 2$

$$Y_A + Y_B + Y_C = 5 \qquad (*)$$

All $\binom{7}{5}$ solutions to $(*)$



Allowed solutions =

$$\binom{7}{5} - \binom{4}{2} - \binom{4}{2} - \binom{3}{1} =$$

$$21 - 6 - 6 - 3 = 6$$

b) $G(x) = \left(x^2 + x^3 + x^4\right) \cdot \left(x^5 + x^6 + x^7 + x^8\right) \cdot$

$\cdot \left(x^3 + x^4 + x^5\right) = x^{10} \dfrac{\left(1-x^3\right)^2}{(1-x)^2} \cdot \dfrac{\left(1-x^4\right)}{(1-x)}$

We are seeking the coefficient in front of x^{15}.

$$G(x) = \frac{x^{10}\left(1 - 2x^3 + x^6\right)\left(1 - x^4\right)}{(1-x)^3}$$

$$\frac{1}{(1-x)^3} = 1 + \binom{3}{1}x + \binom{4}{2}x^2 + \dots + \binom{7}{5}x^5 + \dots$$

so we can get x^{15} in three ways:

1) $x^{10} \cdot \binom{7}{5}x^5$

2) $-2x^{13} \cdot \binom{4}{2}x^2$

3) $-x^{14} \cdot \binom{3}{1}x$.

The coefficient is $\binom{7}{5} - 2\binom{4}{2} - \binom{3}{1} =$

7.

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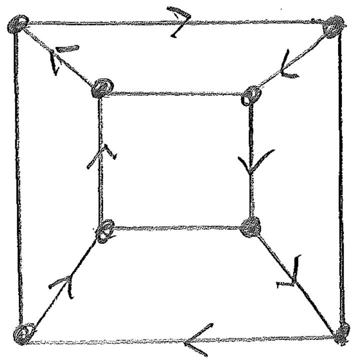
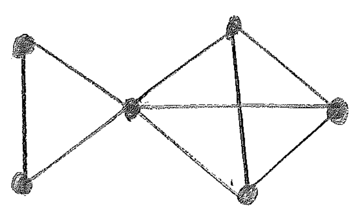
2 a) 12 b) 2 c) 3

Two examples

3 a)



b)



4) $2e = 6 \cdot 3$

$e = 9$

$r - e + V = r - 9 + 6 = 2$

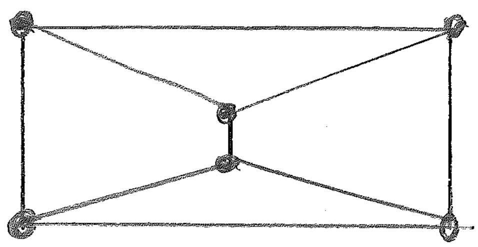
$r = 5$

$2e = 18 = \deg(R_1) + \deg(R_2) + \deg(R_3) + \deg(R_4) + \deg(R_5)$.

Try for example

$18 = 4 + 4 + 4 + 3 + 3$

I take the outer region as a square.



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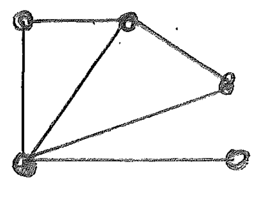
5) Let's try $4, 3, 2, 2, 1$

a)



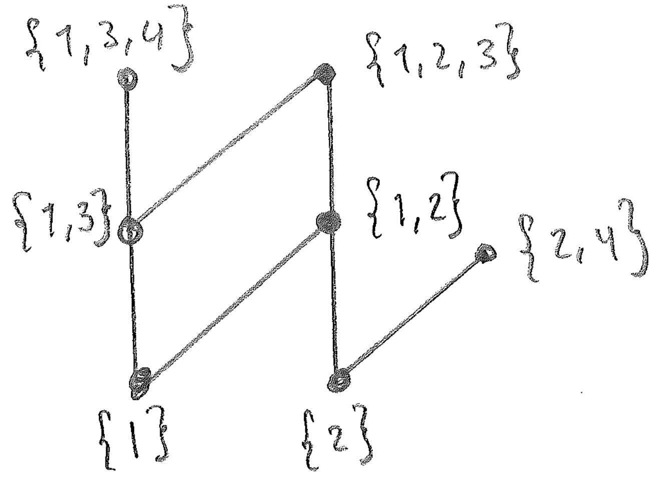
Then

b) $V, V-1, V-2 \dots 1$. Impossible

Highest degree is $V-1$.

3

6 a)



b) aRb means $a-b=k\cdot 3$

bRc means $b-c=l\cdot 3$

Add the two equations

$a-b+b-c=k\cdot 3+l\cdot 3$

$a-c=(k+l)3$ so

aRc. Transitivity is shown!

$[0]=\{\ldots -3,0,3,\ldots\}$

$[1]=\{\ldots -2,1,4,\ldots\}$

$[2]=\{\ldots -1,2,5,\ldots\}$

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7 $V=6$ for $K_{3,3}$

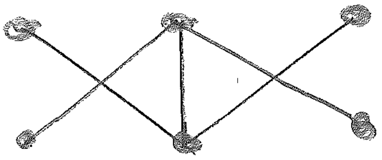
so e is one less, five.

$2e=2\cdot 5=10=\deg(v_1)+\deg(v_2)+\deg(v_3)+\deg(v_4)+\deg(v_5)+\deg(v_6)$.

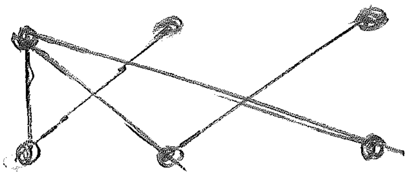
Write 10 with six integers.

Each integer lies between 1 and 3

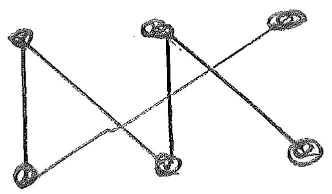
i) $3+3+1+1+1+1$



ii) $3+2+2+1+1+1$



iii) $2+2+2+2+1+1$ A path



b) $3 \cdot 3 = 9$ of type 1 above.

$6 \cdot \binom{3}{2} \cdot 2 = 36$ of type 2

6 choices for degree 3 vertex,
To avoid cycles the 2 degree 2 vertices must be in lower row if degree 3 vertex is in top row.

Also 36 paths of type 3.
$3 \cdot 3 \cdot 2 \cdot 2 = 36$. One degree 1 in top row, one in bottom row.

In total: $9+36+36 = 81$ spanning trees.