Linnaeus University
Mathematics
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Exam in Discrete Mathematics, 1MA462, 7,5 hp
Friday 24th of May 2019, Time 08.00-13.00.

Note: To obtain maximal points a complete solution, presented in such a way that calculations and reasoning are easy to follow, is demanded. If nothing else is given you can assume that all graphs are connected, undirected, loopfree and not multigraphs.

Aid: Sheet with formulas and concepts.

1. 15 votes were lost in an election. They belonged to three political parties, A , B and C. Based on previous elections we can assume

$$2 \leq x_A \leq 4, \quad 5 \leq x_B \leq 8, \quad 3 \leq x_C \leq 5. \tag{1}$$

Here x_A, x_B and x_C denote the number of votes to the three parties, $x_A+x_B+x_C = 15$. In how many ways can the 15 votes be distributed to the three parties with these restrictions?
a) Solve the problem with inclusion-exclusion. (3p)
b) Solve the problem with generating function. (3p)

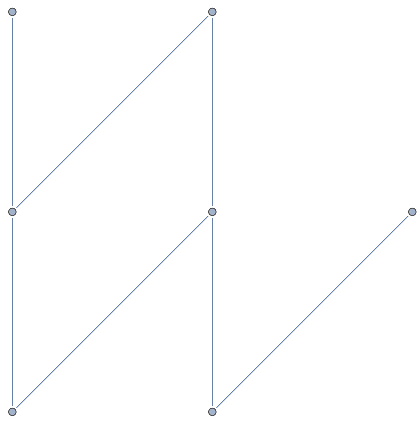
2. The minimum number of colors needed for a proper coloring of a graph G is called the chromatic number and is denoted $\chi(G)$. Give the following chromatic numbers.
a) $\chi(K_{12})$
b) $\chi(K_{15,30})$
c) $\chi(C_{17})$ (3p)

3. Two tasks about Hamilton cycles.
a) Draw a Hamilton cycle in Q_3. Q_3 is the planar graph with 8 vertices where all regions have degree 4 and all vertices have degree 3. (2p)
b) A theorem tells us that if a graph G has v vertices and $\deg(x) + \deg(y) \geq v$ for all non-adjacent pairs of vertices x and y then G has a Hamilton cycle. Draw a graph with 6 vertices and where $\deg(x) + \deg(y) \geq 5$ *without* a Hamilton cycle. Again x and y is any pair of non-adjacent vertices. Non-adjacent vertices have no edge in common. (2p)

4. How many regions have a planar graph with six vertices which is 3-regular, that is all vertices have degree 3? Draw such a graph. (3p)

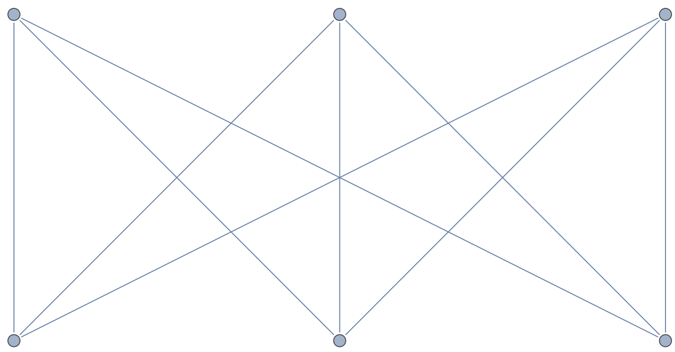
5. The *degree sequence* for a graph is a list of the degrees for the vertices. For example the path P_4 has the degree sequence 2,2,1,1.
a) Draw a connected graph with five vertices and with four different degrees. (2p)
b) Can the v numbers in the degree sequence for a connected graph with v vertices be distinct? With distinct we mean that no two numbers are equal to each other. Motivate your answer. (2p)

6. a) A Hasse diagram is shown below. Unfortunately the labels at the vertices are missing. Give suggestions of what they can be. The relation is the subset relation, that is $A\mathcal{R}B$ if and only if $A \subseteq B$, on the set of all subsets of four elements $\{1, 2, 3, 4\}$. (2p)



b) An equivalence relation on the integers $\mathbb{Z}$ is:
$a\mathcal{R}b$ if and only if $a \equiv b \pmod 3$, $a, b \in \mathbb{Z}$.
Show that the relation is transitive and give the equivalence classes. (2p)

7. A *spanning tree* T to a connected graph G is a subgraph of G which is a tree and contains all the vertices of G. Let, from now on, $G = K_{3,3}$, see figure below.
a) Draw three non-isomorphic spanning trees in $K_{3,3}$ (3p)



b) Let us now also include isomorphic trees. How many spanning trees are there then in total in $K_{3,3}$? (3p)

Good Luck!