

# Discrete Math 11/6 2019

1)

$$3 \cdot 10 \cdot 26 \cdot 26 \cdot 2 = 40560 > \# \text{ students at LNU.}$$

Annotations:

- which digit (points to 3)
- upper case (points to 2)
- position digit (points to 10)
- for lower case (points to 26)
- shift of the two letters (points to 26)

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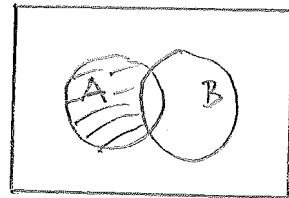
2)

a)  $\binom{17}{14}$

b)



c)



d)  $(1+x)^5$

e)  $n$  must be odd.  
Then  $\deg(x)$  is even for all  $x \in V$ .

f)  $K_{n,n}$   
 $n \geq 2$ .

g)  $5 \cdot 4 \cdot 3 = 60$

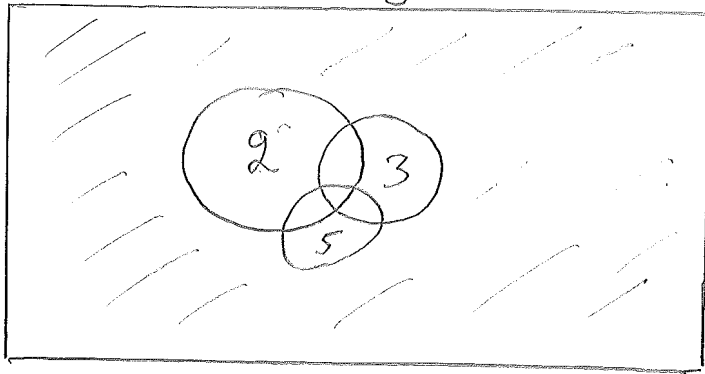
h)  $2^7$



j)  $2^8 = 256$ .

3)

All 60 integers



(2) means  
multiples of  
2 etcetera,

$$60 - \left( \underset{\substack{\uparrow \\ \text{even} \\ \text{numbers}}}{30 + 20 + 12} \right) + \left( \underset{\substack{\uparrow \\ \text{multiples} \\ \text{of } 6}}{10 + 6 + 4} \right) - \underset{\substack{\uparrow \\ 30 \text{ and} \\ 60}}{2}$$

$$= 16 \quad (14 \text{ primes and } 1 \text{ and } 7^2 = 49)$$

4)

4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4
	1	2	3	4

The products  
are given in the  
grid.

Classes with only 1 element:  $[(4,4)]$  and  $[(1,1)]$   
and  $[(3,3)]$ .

Classes with two elements:

$[(1,2)]$ ,  $[(1,3)]$ ,  $[(2,3)]$ ,  $[(2,4)]$ ,  $[(3,4)]$ .

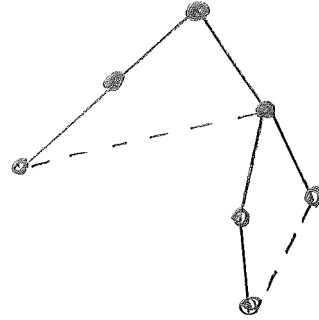
One class with 3 members:

$[(2,2)] = \{(2,2), (1,4), (4,1)\}$ .

$$3 \cdot 1 + 5 \cdot 2 + 1 \cdot 3 = 16 \quad \text{OK!}$$

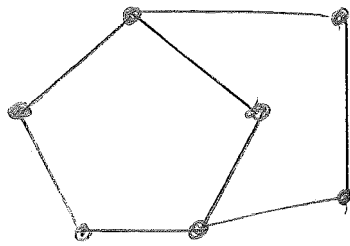
5)

a) For trees  $e = v - 1$ . Adding two edges creates two cycles.



b)  $\frac{v+1}{2}$  ( $v$  odd)  
 $\frac{v}{2}$  ( $v$  even)

c)  $\lfloor \frac{16}{3} \rfloor = 5$



Two cycles of length 5.  
 One cycle of length 6.

6)

a)  $2e = \sum_i \deg(R_i) \geq 3 + 3 + 3 + \dots + 3 = 3r$ . So  $r \leq \frac{2e}{3}$ .

$r = e - v + 2$  (Euler's formula)

so  $e - v + 2 \leq \frac{2e}{3}$

$e \leq 3(v - 2)$  for planar graphs.

6b)  $K_{11}$  has  $\binom{11}{2} = 55$  edges.

If  $v=11$   $G$  can be planar only if  $e \leq 3(11-2) = 27$ .

$$55 - 27 = 28.$$

So if  $G$  has 27 edges then  $\bar{G}$  has 28 edges.

If  $G$  has  $e$  edges,  
 $\bar{G}$  has  $55 - e$  edges.

$e$  or  $55 - e$  is  $\geq 28$ .