Discrete Math 29/8 2019

1)

$$X_1 + X_2 + X_3 = 25$$

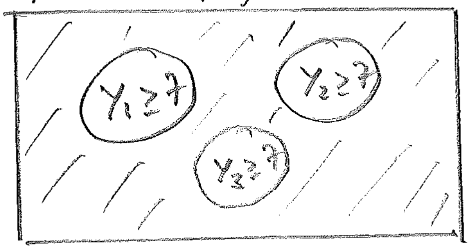
$$5 \le X_i \le 11, \quad i = 1, 2, 3$$

with $X_i = 5 + Y_i$ we get

$$Y_1 + Y_2 + Y_3 = 10 \qquad (*)$$

$$0 \le Y_i \le 6$$

All $\binom{12}{2} = 66$ solutions to $(*)$



$Y_1 \ge 7$: $\quad Y_1 = 7 + Z_1, \quad Z_1 + Y_2 + Y_3 = 3$

has $\binom{5}{2} = 10$ solutions.

same for $Y_2 \ge 7$ and $Y_3 \ge 7$

So in total $66 - 3 \cdot 10 = 36$ solutions.

GF: $\quad G(x) = \left(x^5 + x^6 + \dots x^{11}\right)^3 =$

$$x^{15} \frac{\left(1 - x^7\right)^3}{(1 - x)^3}.$$

Search coefficient in front of x^{25}.

2) K_n is bipartite for $n=2$.
C_n is bipartite for even $n=$
$4, 6, 8, \ldots$

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3) a) Reflexive? $x|x$ yes

Anti-symmetric? $x|y \Rightarrow y = k \cdot x$
$y|x \Rightarrow x = l \cdot y$

so $y = k \cdot l \cdot y \Rightarrow k \cdot l = 1$, $k, l \in \mathbb{Z}$
so $k = l = 1$ and therefore $y = x$.
Yes anti-symmetric.

Transitive? $x|y \Rightarrow y = k \cdot x$
$y|z \Rightarrow z = m \cdot y$

so $z = m \cdot y = m \cdot k \cdot x \Rightarrow x|z$. Yes

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b) reflexive? $x - x = 0 \cdot 7$ yes

symmetric? if $x - y = k \cdot 7$
$y - x = (-k) \cdot 7$. Yes

transitive? $y - x = k \cdot 7$ (1)
$z - y = n \cdot 7$ (2)

(1) + (2): $z - x = (k+n) \cdot 7$ so $x R z$.

4) a) 3 N-moves and 3 E-moves.

ENNENE is an example

$\binom{6}{3}$ such words (paths).

b) 3D: 1 path

2D: $\frac{4!}{2!1!1!} = 12$ paths

DNED for example.

1D: $\frac{5!}{1!2!2!} = 30$ paths

ENDEN for example.

In total 43 paths.

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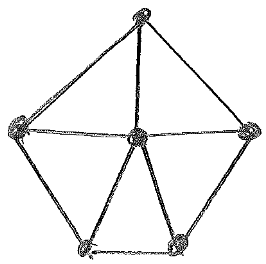
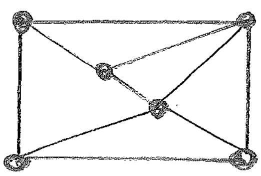
5. $6 - 10 + V = 2$ according to Euler,

$V = 6$

$20 = \sum \deg(R_i) = 5 + 5 \cdot 3$



$= 4 \cdot 3 + 4 + 4$

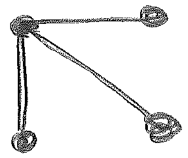


is another example.

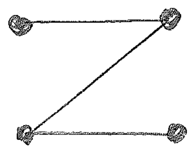
6) a) K_4 has 4 vertices
so the tree has 3 edges.

For T: $2 \cdot 3 = 6 = \deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4)$

$6 = 3+1+1+1$



$6 = 2+2+1+1$

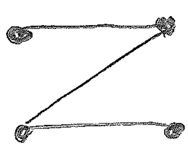
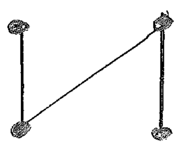


b) 4 of the $3+1+1+1$ type.

$+$ 12 of the $2+2+1+1$ type.

$$\begin{array}{r} 4 \\ +\ 12 \\ \hline 16 \end{array}$$

$\binom{4}{2} = 6$ ways to select the degree 1 vertices.
Then 2 different paths to join them.

Like  and 

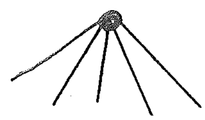
7) The complete graph K_v has $\frac{v(v-1)}{2}$ edges.

The difference is

$$\frac{v(v-1)}{2} - \frac{(v-1)(v-2)}{2} = \frac{(v-1)}{2}\left[v-(v-2)\right]$$

$= v-1 =$ degree of each vertex in K_v. So if $e > \frac{(v-1)(v-2)}{2}$ the difference is smaller than the degree of the vertices in $K_v \Rightarrow$ No vertex can be cut off.

For $v=6$



If $e = 11$ only 4 edges to take away, K_6 has 15 edges.

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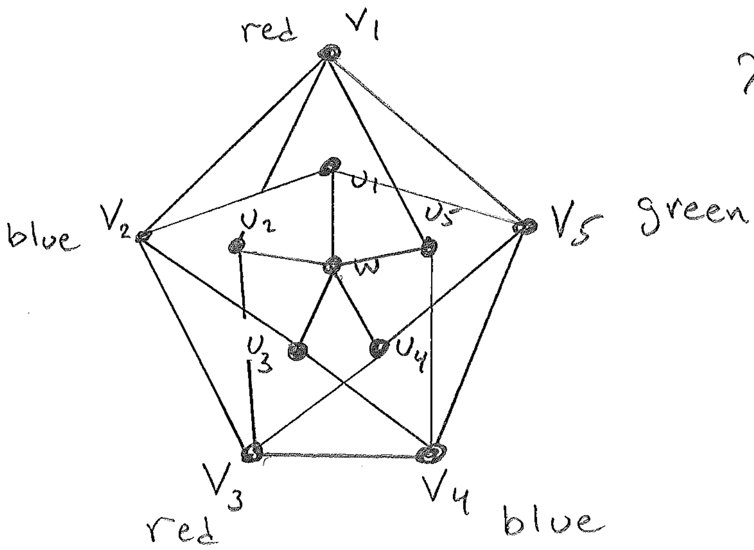
8)



$\chi(C_5) = 3$

u_1 – red

u_5 – green

u_4 – blue

So a fourth color is needed for w.