Linnaeus University
Mathematics
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Exam in Discrete Mathematics, 1MA462, 7,5 hp
Thursday 29th of August 2019, Time 08.00-13.00.

Note: To obtain maximal points a complete solution, presented in such a way that calculations and reasoning are easy to follow, is demanded. If nothing else is given you can assume that all graphs are undirected, connected, loopfree and not multigraphs.

Aid: Sheet with formulas and concepts.

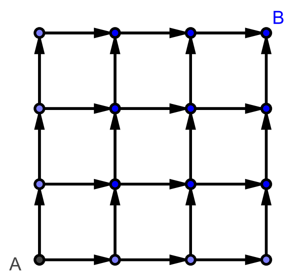
1. You have to buy 25 bagels. They are of three types: plain, tomato and blueberry. The restrictions are: At least five of each sort but not more than 11 of any sort.
   In how many ways can you do this? Solve the problem with inclusion and exclusion or by using generating function. (4p)

2. For which values of $n \geq 2$ is K_n bipartite? For which values of $n \geq 3$ is C_n bipartite? (2p)

3. a) Show that the divides relation is a partial order on the positive integers. The divides relation, $\mathcal{R}$, is defined by: $x\mathcal{R}y$ if and only if $x|y$. Here x and y are positive integers. (3p)

   b) Define the relation $\mathcal{R}$ on the the integers $\mathbb{Z}$ by $x\mathcal{R}y$, for $x, y \in \mathbb{Z}$, if and only if $x \equiv y \pmod 7$. Show that this is an equivalence relation on $\mathbb{Z}$. (3p)

4. A bicyclist is going from $A = (0, 0)$ to $B = (3, 3)$. She is only allowed to go north and east on the one-way streets.

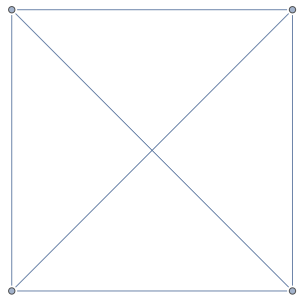


   a) How many paths are there for the bicyclist going from A to B? (2p)

   b) Let us now allow for diagonal moves such that $(x, y) \to (x + 1, y + 1)$ is possible. How many paths are there now for the bicyclist going from A to B if she uses *at least* one diagonal move? Here you can use the sum rule and sum over the number of diagonal moves. (3p)

5. A planar graph has ten edges and six regions. How many vertices has the graph? Draw such a graph. (3p)

6. A spanning tree T to a graph G is a connected subgraph of G which is a tree and contains all the vertices in G. Let now $G = K_4$, see figure below.



a) Draw two non-isomorphic spanning trees in K_4. (2p)

b) Let us now also include isomorphic trees. How many spanning trees are there then in total in K_4? (2p)

7. Show that a simple graph with v vertices is connected if it has more than $\frac{(v-1)(v-2)}{2}$ edges. (3p)

8. Is it possible to find a simple connected graph, G, with chromatic number $\chi(G) = 4$ *without* any triangles, C_3, as subgraphs? Yes, here comes an example with $v = 11$ and $e = 20$: Start with a C_5 with vertices $v_1, v_2, \cdots, v_5$. Inside the cycle you draw a star, $K_{1,5}$. The vertex in the center of the star we denote w and the five others are denoted $u_1, u_2, \cdots, u_5$. Make sure that v_1 is close to u_1 et cetera. Ten more edges to draw and they are obtained in such a way that for each edge v_iv_j two edges u_iv_j and v_iu_j are drawn. Now you can draw the graph, G, we are seeking. Finally, explain why the chromatic number must be 4 for this graph. Hint: make first a proper coloring of the subgraph $G - w$ (the graph with vertex w removed and all edges attached to it): (3p)

Good luck!