Questions and Answers on Graphs and Trees. The numbers refer to 6:th edition of Rosen's book.

9.2.46

G is a graph with v vertices and e edges. Let M be the maximum degree of the vertices and m the minimum degree. Show that $M \geq 2e/v \geq m$.

Student: Don't understand.
Teacher: Remember 2e=sum of all degrees and then this sum is estimated above and below.

9.5.9

T: Adding new bridges between B and C and B and D doesn't make the Euler circuit possible i Köningsberg. Deg(A) is still equal to 5.

About isomorphism

T: If you have to check if two graphs are isomorphic to each other start with the most obvious things. The number of edges, vertices and vertices with a certain degree must be the same. Then you go one with the number of cycles of a given (short) length. See my notes. If you get the same numbers for the two graphs it is time to find the bijection between the two sets of vertices. Adjacent matrix can be used to check that you have found an isomorphism, see page 617. The two matrices must then be equal.

9.7.5

Determine if the graph is planar.

$E=\{(a,b),(a,c),(a,e),(b,f),(b,d),(d,c),(d,e),(e,f),(c,f)\}$.

S: The number of edges is $\leq 3v-6$ but the graph seems to be non planar. Is this correct?
T: Exactly, it can be like that. Think of K 3,3. The inequality is a necessary condition but it is not a sufficient one. For K 3,3 you can improve the inequality by using the fact that the shortest cycle in a bipartite graph is of length four. Try to derive it! Using this new improved inequality you will see that K 3,3 is nonplanar.

9.7.21

Determine whether the given graph is homeomorphic to K 3,3.

E={(a,g),(a,d),(d,g),(d,c), (g,b),(g,e),(e,b),(b,h),(e,h),(h,c),(c,f),(h,f)}

T: Since deg(g)=deg(h)=4 it can not be homemorphic to K3,3. The new included vertices have degree 2. In K 3,3 all vertices have degree 3.

9.7.23

Is the given graph planar?

E={(a,e),(a,f),(a,h),(b,e), (b,f),(b,g),(c,f),(c,g),(c,h),(d,e),(d,g),(d,h)}

T: Try first to draw it planar. I started with a Hamilton cycle (octagon) and then it is only 4 edges left to draw and it is possible to draw them planar.
If you fail you suspect it is nonplanar and then you use Kuratowski's theorem. You have to try and see if you can find a subgraph homeomorphic to K5 or K 3,3.

9.8.17

Scheduling of final exams

T: Five different time slots is minimum. The order of the exams can differ.

10.1.16

Which complete bipartite graphs Km,n are trees?

T: For trees: v=e+1. For bipartite graphs: e=m*n, v=m+n. So

$0= e-v+1= m*n – m – n + 1 = (m-1)*(n-1)$

m or n or both must be equal to 1.

10.1.33

How many different isomers do C_3H_8 have? Or how many nonisomorphic trees can you find if 3 vertices have degree 4 and the other 8 vertices have degree 1?

S: Is the answer a straight carbon chain with 3,2,3 hydrogen atoms attached to the carnon atoms?
T: Yes it is a tree. Since v=11, 2e=3*4+8*1=20 so e=10. Thus v=e+1 which always holds for trees.
Start with C as a root. 1 or 2 C on next level but these two trees are isomorphic to each other.

10.1.11a, 10.11a, 10.11.13a

S: In regards to unrooted nonisomorphic trees, I mean problem # 11a and #13 a in section 10.1 page 694. How are they coming up with 1 for 11a and 3 for 13a? I understand how they come up with rooted but not sure how to classify unrooted nonisomorphic trees.
Also for the second question I refer to your lecture notes part III. You have a table for the directed path on the first page and for the step 2 path a,c you have 2. Is that the correct number or is it suppose to be 3? If it is 2 why?

T: Three vertices mean 2 edges and there is only one such graph (unrooted tree). Namely V-shape. If you start to consider about roots then we have 2 (V upside down and a vertical path)
5 vertices means 4 edges. 2*4=8=sum of degrees. 4 is the maximal degree. Must be 5 terms 8=4+1+1+1+1, 8=3+2+1+1+1, 8=2+2+2+1+1. Can you draw the trees which corresponds to each of these sums? Sometimes little artitmetic can guide us! If we consider the rooted ones we start with the 3 above and consider different vertices as roots.

The table is right. The minimum sum is STILL 2 after vertex c is included. This is the idea behind the algorithm. Choosing vertex e directly gives 4 but 3<4 so we take c. We have to proceed carefully so we don't miss a "cheap" way. Think of these websites like MrJet which find cheapest flights (even if it takes strange ways, like going via London to Prague from Copenhagen).