Questions and Answers on Relations. The numbers refer to 6:th edition of Rosen's book.

8.1.1f

aRb iff lcm(a,b)=2. A={0,1,2,3,4} and B={0,1,2,3}

Student: How to think?
Teacher: No 3's and 4's can be in the lcm(a,b)=2 relation. So (1,2), (2,1), (2,2) are the only possibilities.

8.1.3e

S: Why transitive when only (1,1), (2,2), (3,3), (4,4) is in the relation?
T: According to definition of transitivity we have to say that the relation is transitive.

8.1.7 e

xRy iff x is a multiple of y. The set is Z, the intergers.

S: Why not antisymmetric? And in d, how to think?
T: Often it is something tricky about zero or negative numbers. Here the latter. For example 2R(-2) and (-2)R2 so R is not antisymmetric. In d xRy iff x-y is an integer times 7. A standard example of an equivalence relation! Try to show transitivity (hardest part),
xRy -> x-y=k*7
yRz -> y-z=l*7
Is x-z a multiple of 7? (Here k and l are integers)

8.1.41

How many of the 16 different relations on {0,1} contain the pair (0,1)?

S: Don't understand!
T: A={0,1}. AxA has 4 elements. Thus 2^4=16 relations on A. Include or not include in R. To choices for each element. 2*2*2*2=16. Half of them includes (0,1).

8.3.9a

R={(a,b)|a>b} is the relation on A. A={1,2,3,….., 99, 100}. How many ones in the 0-1 matrix?

T: Matrix M is 100x100. Element M_ab=1 if aRb otherwise 0..
First row in M contain 99 1's, second row 98 etc. So in total 1+2+3+....+98+ 99=50*99=4950 nonzero entries.

8.3.7
Determine from 0-1matrix wheater a relation is transitive, symmetric..

S: How do you see in the matrix if a relation is transitive?
T: See section 8.4 for the details and also my lectures. For small sets you can draw the digraph and check the various possibilities.

8.3.15
Find the matrices for RoR=R^2, R^3 etc

S: I get 2 in the matrix multiplication. I interpret them as 1. Right?
T: Right! Called Coolean product and you can stop when you get 1*1. Think of air flights. If you can travel from C (openhagen) to M (ilan) with one stop at P (aris). C R P and P R M. Then C R^2 M.

8.5.3b
The relation on functions from Z to Z is f g R iff f(0)=g(0) or f(1)=g(1). Is this an equivalence relation?

T: 3 things to check. Reflexive? Symmetric? Transitive?
The relation in b is symmetric and reflexive but not transitive.
Take 3 functions f, g and h where
f(0)=g(0)=1 och h(0)=2
f(1)=3 och g(1)=h(1)=4.
fRg and gRh but f is not related to h. Then R is not transitive.

8.5.15
(a,b)R(c,d) iff a+d=b+c. Show it is an equivalence relation. A is here the set of pairs of positive integers.

T: Transitivity is the hardest part. (a,b)R(c,d) and (c,d)R(e,f) so a+d=b+c and c+f=d+e. Add the 2 equations. Is a+f=b+e?

8.6.3b

T: a is smaller or equal to b. This is not a poset. Two different persons can have the same length.

8.6.19

Find the lexicographic ordering of bit strings 0, 01, 11, 001, 010, 011, 0001 and 0101 based on the ordering 0<1.

S: Why comes 01 before 010?
T: Replace 0->a and 1->b. ab comes before aba in a dictionary.