Linnaeus University
Mathematics
Hans Frisk

Formulas and text to be used at exam in discrete mathematics, 1MA462, 7,5 hp.

1 Combinatorics

To pick r balls from a bowl with n balls can be done in the following number of ways:

$$n^r \quad \text{with repetitions and with respect to order}$$

$$\frac{n!}{(n-r)!} \quad \text{without repetitions and with respect to order}$$

$$\frac{n!}{(n-r)!r!} = \binom{n}{r} \quad \text{without repetitions and without respect to order}$$

$$\frac{(n-1+r)!}{(n-1)!r!} = \binom{n-1+r}{r} \quad \text{with repetitions and without respect to order}$$

2 Some Generating Functions

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + x^n$$

$$\frac{1-x^{n+1}}{1-x} = 1 + x + x^2 + \cdots + x^n$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\frac{1}{(1-x)^n} = 1 + \binom{n}{1}x + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots$$

3 Relations

A binary relation on a set A is a collection of orded pairs of elements of A. In other words, it is a subset of the Cartesian product $A^2 = A \times A$. More generally, a binary relation between two sets A and B is a subset of $A \times B$.

A binary relation is the special case $n = 2$ of an n-ary relation $R \subseteq A_1 \times A_2 \times \cdots \times A_n$, that is, a set of n-tuples where the jth component of each n-tuple is taken from the jth domain A_j of the relation.

Some important types of binary relations over a set X are:
Reflexive: for all x in X it holds that xRx.
Symmetric: for all x and y in X it holds that if xRy then yRx.
Antisymmetric: for all distinct x and y in X, if xRy then not yRx.
Transitive: for all x, y and z in X it holds that if xRy and yRz then xRz.

A *lattice* is a partially ordered set in which every two elements have a unique *supremum* (also called least upper bound) and a unique *infimum* (also called greatest lower bound). An example is given by the positive integers, partially ordered by divisibility, for which the unique supremum is the least common multiple and the unique infimum is the greatest common divisor.

4 Graph Theory

A graph G consists of two types of elements, namely vertices and edges. Every edge has two endpoints in the set of vertices, and is said to connect or join the two endpoints. An edge can thus be defined as a set of two vertices (or an ordered pair, in the case of a directed graph). The two endpoints of an edge are also said to be adjacent to each other.

Alternative models of graphs exist. For example, a graph may be thought of as a square (0,1)-matrix.

A vertex is simply drawn as a node or a dot. The vertex set of G is usually denoted by $V(G)$, or V when there is no danger of confusion. The order of a graph is the number of its vertices, $|V(G)|$.

An edge (a set of two elements) is drawn as a line connecting two vertices, called endpoints or endvertices. An edge with endvertices x and y is denoted by xy (without any symbol in between). The edge set of G is usually denoted by $E(G)$, or E when there is no danger of confusion.

A loop is an edge whose endpoints are the same vertex. An edge is multiple if there is another edge with the same endvertices; otherwise it is simple. A graph is a simple graph if it has no multiple edges or loops, a multigraph if it has multiple edges, but no loops. When stated without any qualification, a graph is usually assumed to be simple.

A subgraph of a graph $G = (V, E)$ is a graph $H = (W, F)$, where $W \subseteq V$ and $F \subseteq E$.

The complement of a graph, $\overline{G}$, is a graph with the same vertex set as G but with an edge set such that xy is an edge in $\overline{G}$ if and only if xy is not an edge in G.

Two graphs G and H are said to be *isomorphic*, denoted by $G \sim H$, if there is a one-to-one correspondence between the vertices of the graph such that two vertices are adjacent in G if and only if their corresponding vertices are adjacent in H. A graph G is said to be *homeomorphic* to a graph H if they can be obtained from the same graph by a sequence of *elementary subdivisions*. When doing an elementary subdivision you remove an edge $\{u, v\}$ and then adding a vertex w and the edges $\{u, w\}$ and $\{w, v\}$.

A trail or circuit is Eulerian if it uses all edges precisely once.

A simple path or cycle is Hamiltonian if it uses all vertices exactly once.

A tree is a connected simple graph without cycles.

In a *proper coloring* of a graph no two adjacent vertices are assigned the same color.

A vertex of a rooted tree is called a $leaf$ if it has no children. Vertices that have children are called $internal\ vertices$. The tree is called a $full\ m-ary\ \ tree$ if every internal vertex has exactly m children. An m-ary tree with $m = 2$ is called a binary tree.