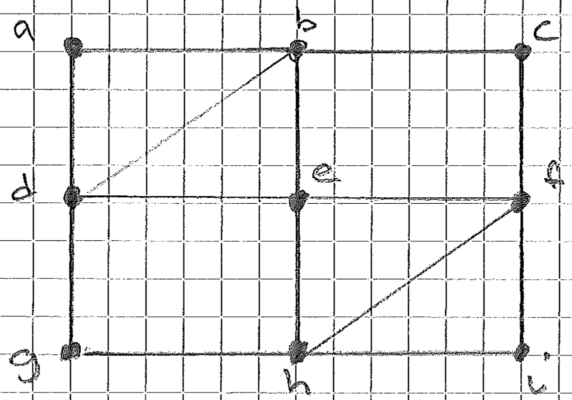
Introduction

The first chapters are about Logic, Sets, Algorithms, Induction and Counting.

Let's see how these subjects appear in graph theory.

Ex) Corners and Roads



Can a postman travel over all roads once and come back to the same point?

Such a circuit is called an Euler circuit (EC).

Instead of corners and roads we will speak about vertices and edges

A graph $G = G(V, E)$

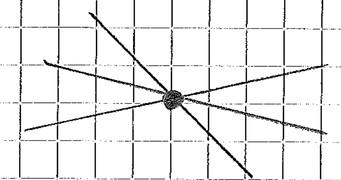
$V = \{a, b, c, d, e, f, g, h, i\}$ Vertex set.

$E = \{(a,b), (a,d) \ldots \ldots (h,i)\}$ Edge set.

G has an EC $\longrightarrow$ even number of roads meet at all corners. Or, $\deg(v)$ even for $\forall v \in V$.

$\deg(a) = 2 = \deg(g) = \deg(c) = \deg(i)$.
$\deg(b) = \deg(d) = \deg(e) = \deg(f) = \deg(h) = 4$

A <u>necessary condition</u> for the existence of an EC.

Proof: 

$2+2+2=6$

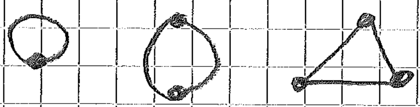
But surprisingly it also a <u>sufficient condition</u> for an EC!

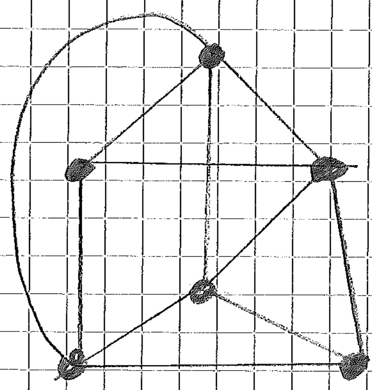
G has an EC. $\Longleftrightarrow$ $\deg(v) = 2k \quad \forall v \in V,\ k \in \mathbb{Z}^+$

"if and only if"

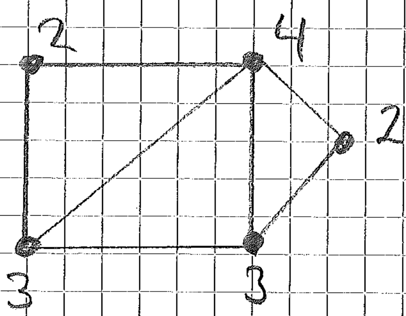
How to prove $\Longleftarrow$?

For example by an
<u>induction proof</u> over the
number of edges.

Start with few edges
 and check them.
Assume there are EC up to
k edges ....

Ex)

Is there an EC?

Hamilton cycle (HC):
Visit all vertices <u>once</u>
and come back to the
same vertex.

Ex)


Is there a HC?

$|V| = 5 = n$

A sufficient condition for a HC:

$$\deg(u) + \deg(v) \geq n$$

for all pair of vertices u, v without an edge in common.

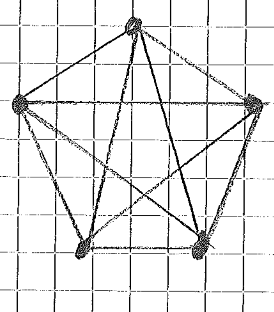
But it is not a necessary condition!

$2 + 2 < 5$ but we found a HC!

How to find a HC in a big graph? We need an algorithm!

How will computing time grow with the number of vertices?

Ex)



How many Hamilton cycles are there?