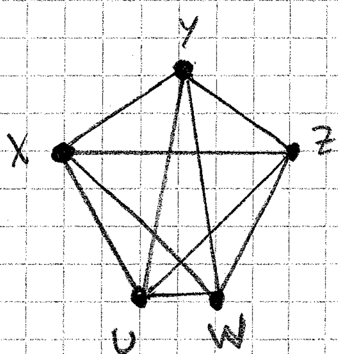
Graphs & Trees, part I

Def: $G = (V, E)$. V is a set of <u>vertices</u> (or nodes). E is a set of <u>edges</u>. Often $E \subseteq V \times V$.

Ex)

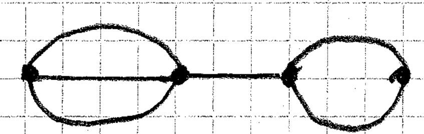


$|V| = 5$

$|E| = 10$

This is a <u>simple</u> graph (only one edge between 2 vertices).

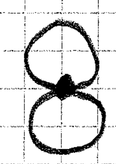
<u>Multigraph</u>



$|V| = 4$

$|E| = 6$

Loops

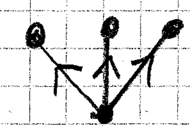


$|V| = 1$

$|E| = 2$

The graphs above are <u>undirected</u>.

<u>Digraph</u>



$|V| = 4$

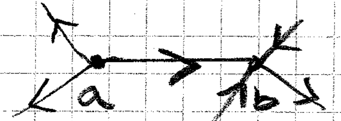
$|E| = 3$

and it is simple.

Ex) WWW $|V| = 3 \cdot 10^9$ (1999)

$|E| = 20 \cdot 10^9$

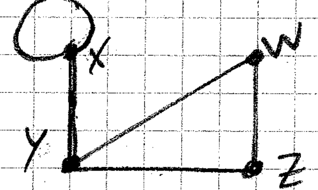
one vertex for each web page.



a links to b.

9.2 Graph Terminology and Special Types of Graphs. (Mostly for undirected graphs)

G_1



$|V| = 4$

$|E| = 5 = e$

y and w are <u>adjacent</u>. The edge $\{y, w\}$ is incident with y and w.

<u>Degree of a vertex</u>, deg(v).

deg (x) = 3 note!

deg (y) = 3

deg (w) = deg (z) = 2

Theorem: $$2e = \sum_{v \in V} \deg(v)$$

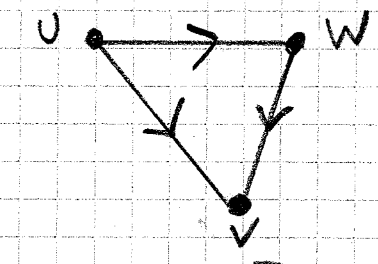
For the G_1 above

$$2 \cdot 5 = 3 + 3 + 2 + 2$$

Always an even number of vertices with odd degree. !

G_2

Digraph.



u is initial vertex to $\{u, v\}$.
v is terminal to $\{u, v\}$.

$\deg^-(u) = 0$ $\quad$ $\deg^-(v) = 2$ $\quad$ $\deg^-(w) =$

$\deg^+(u) = 2$ $\quad$ $\deg^+(v) = 0$ $\quad$ $\deg^+(w) = 1$

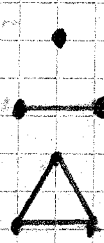
$$\sum_{v_i \in V} \deg^-(v_i) = 0 + 2 + 1 = 3 = |E|$$

$$\sum_{v_i \in V} \deg^+(v_i) = 2 + 0 + 1 = 3 = |E|$$

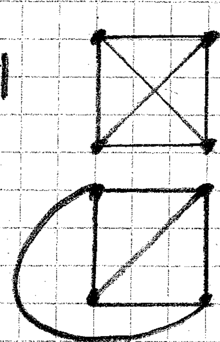
Special graphs

Complete graphs

K_1 $\quad$ K_2 $\quad$ K_3

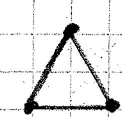


K_4

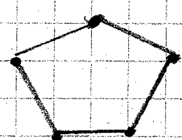


Cycles

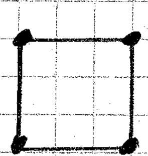
C_3



C_5

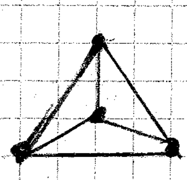


C_4

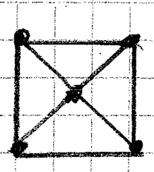


Wheels

W_3

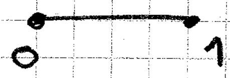


W_4

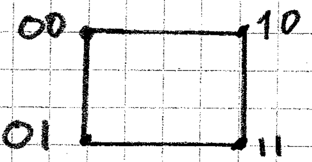


Hypercubes ;

Q_1



Q_2

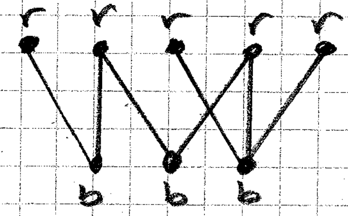


Q_3 is the cube !

Bipartite graphs

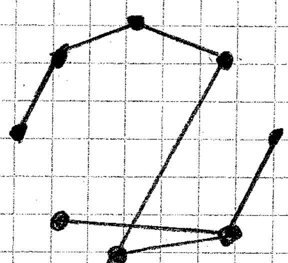
V_1

V_2



$|V| = 5+3 = 8$
$|E| = 7$

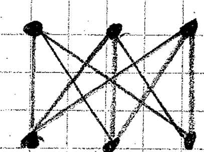
If you can color the vertices of a graph with only 2 colors, red and blue, in such a way that no two adjacent vertices have the same color then the graph is bipartite



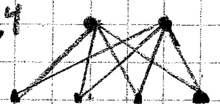
Is this graph bipartite ?

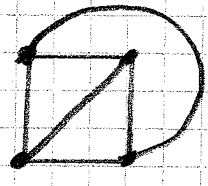
Complete bipartite graph

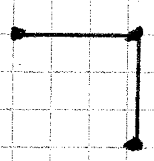
$K_{3,3}$



$K_{2,4}$



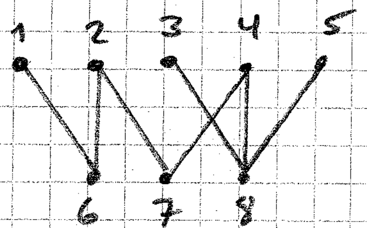
Ex) A <u>subgraph</u> of K_4  (V, E)

is  (W, F)

since
$W \subseteq V$
$F \subseteq E$

9.3 Representing Graphs and Graph Isomorphism

Ex)



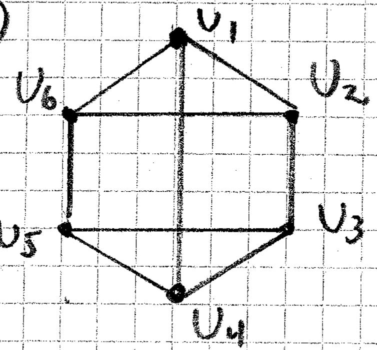
$$\left[\begin{array}{ccccc|ccc} 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&1\\ 0&0&0&0&0&0&0&1\\ \hline 1&1&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 0&0&1&1&1&0&0&0 \end{array}\right]$$

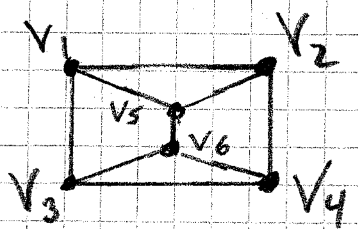
Adjacency matrix

Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are <u>isomorphic</u> if there is a bijection f from $V_1 \xrightarrow{f} V_2$ such that

$$\{a, b\} \in E_1 \iff \{f(a), f(b)\} \in E_2$$

Ex)





Isomorphic?

$f(U_1) =$

$f(U_4) =$