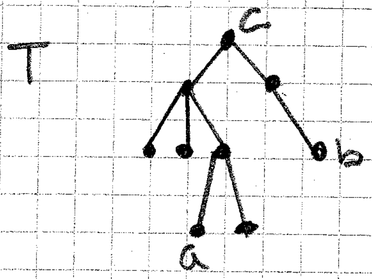
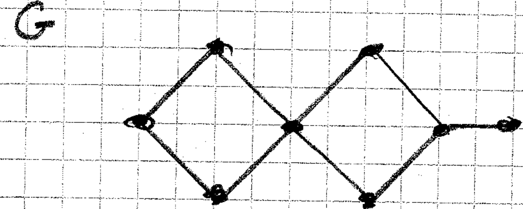
Graphs & Trees, part V.

10.1 **DEF:** A tree is a connected undirected graph with no simple circuits (or cycles)



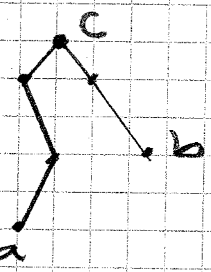


Theorem:

G is a tree

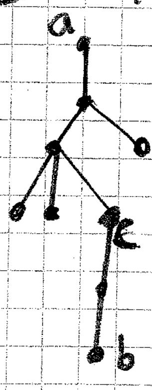
$\Longleftrightarrow$

There is a unique simple path between any pair of vertices.



T above has root c

Here is same T with root a



DEF: An <u>m-ary tree</u> is a rooted tree where every <u>internal</u> vertex has no more than m children.

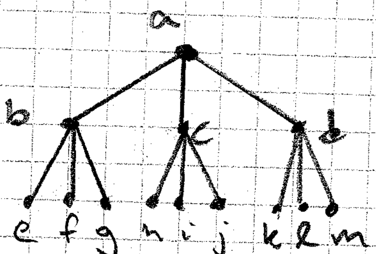
Ex) A <u>full</u> 3-ary tree

level 0

level 1

level 2

height 2



a - root

a, b, c, d - internal vertices

e, f, g ... m are <u>leaves</u>

e, f, g are <u>children</u> of b

<u>height</u> of a rooted tree T = maximum of the levels

<u>Some general results</u>:

n = # vertices, i = # internal vertices, l = # leaves. $n = 13$, $i = 4$, $l = 9$ in the example above

Theorem:  $n = e + 1$. Proof by induction

Theorem: $n = mi + 1$ for a full m-ary tree. $13 = 3 \cdot 4 + 1$

Since $n = l + i$ $\quad 13 = 1 + 3 + 3^2 =$

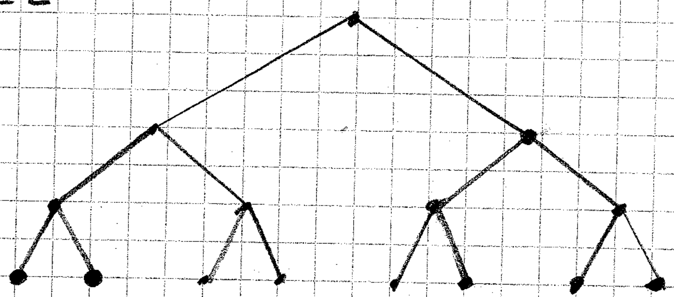
$l = (m-1)i + 1 \quad 1 + 3(1+3) = 1 + 3i$

or $\quad i = \dfrac{(l-1)}{m-1}$

If $m=2$ T is a binary tree,
Then $i = l-1$

Ex) $m=2$

$l=8$
$i=7$



Ex) A table tennis tournament has 1024 participants. How many games have been played after the final game?

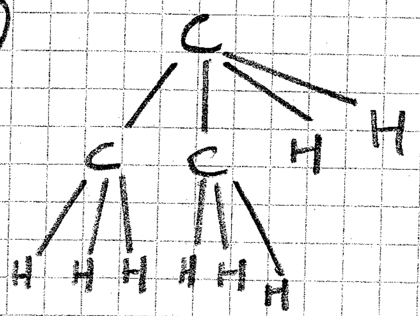
10.1.33 a) C_3H_8 How many isomers?

$\deg(C)=4$
$\deg(H)=1$

$v = 3+8$
$2e = 3\cdot 4 + 8\cdot 1 = 20$
$e = 10 = v-1$ A tree!

How many non-isomorphic trees are there?

1)



2)