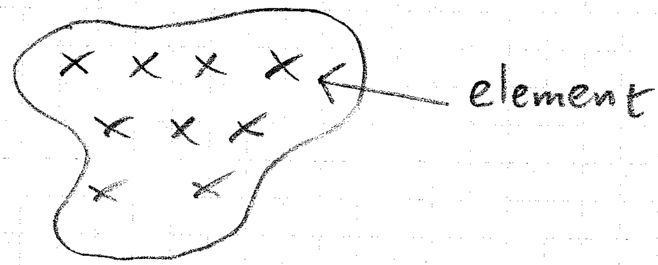
Chapter 2, Sets and Functions

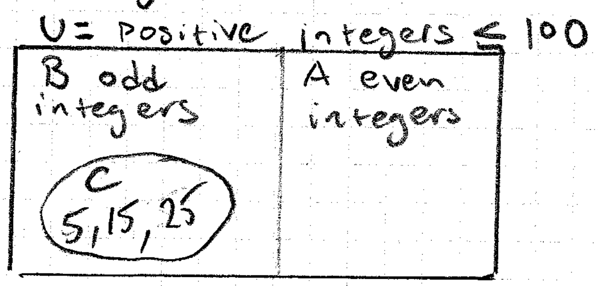
Set – a collection of unorded objects.
Cantor 1895



$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$
$\mathbb{Q} = \{ p/q \mid p \in \mathbb{Z} \land q \in \mathbb{Z} \land q \neq 0 \}$
$\mathbb{R}$ is the set of real numbers

Two sets A and B are equal, $A = B$, if and only if $\forall x (x \in A \Leftrightarrow x \in B)$

Venn diagrams



$B \subseteq U$
$A \subseteq U$
$A \cap B = \emptyset$
$C \subseteq B$
$C \cap B =$
$A \cup B =$

Power set, $P(S)$, of a set S is the set of all subsets to S.

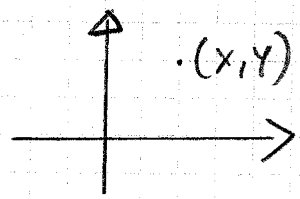
$$P(\{0,1\}) = \{\phi, \{0\}, \{1\}, \{0,1\}\}$$

$|S| = 2$

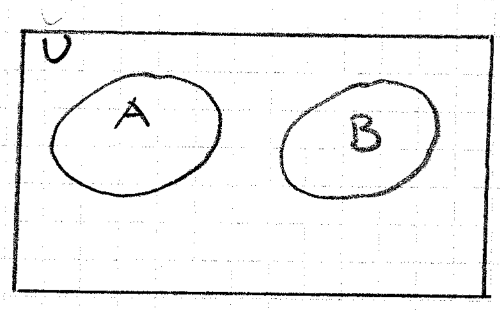
$|P(S)| = 2^2 = 4$

<u>Cartesian products</u>

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

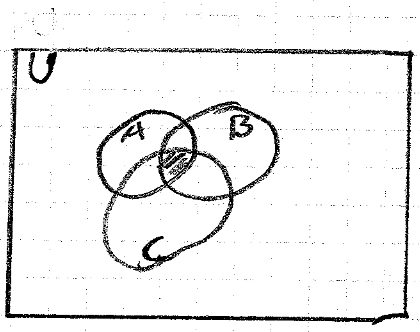


$\mathbb{R}^2$



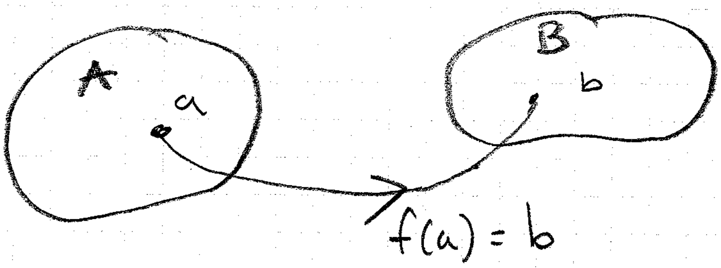
A and B are <u>disjoint</u> sets.

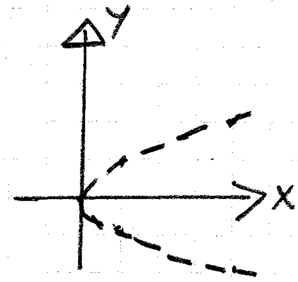
$\bar{A} = U - A$, the <u>complement</u> to A.



The shaded region is $A \cap B \cap C$

Functions (2,3)





NOT A FUNCTION
y(x)

f one-to-one $\iff$ $f(a) = f(b) \implies a = b$

f onto $\iff$ $\forall b \, \exists a \, (f(a) = b)$

Ex) $f: \mathbb{Z}^+ \to \mathbb{Z}^+$, $f(n) = 2^n$

f is one-to-one but not onto. The <u>range</u> of f is $\{2, 4, 8, 16, \ldots\}$.

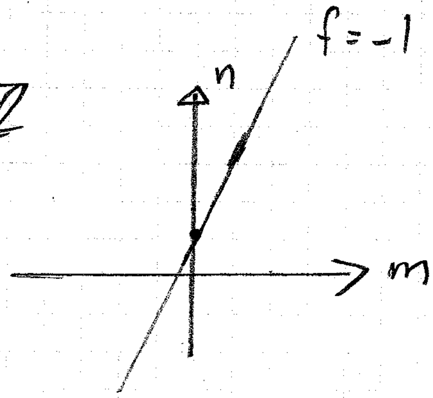
$\lfloor x \rfloor$, floor function. Gives the largest integer $\leq x$. $\lfloor -\pi \rfloor =$

$\lceil x \rceil$, ceiling function. Gives the smallest integer $\geq x$. $\lceil \sqrt{3} \rceil =$

2.2.14 $f: \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z}$

a) $f(m,n) = 2m - n$

onto!



b) $f(m,n) = m^2 - n^2 = (m-n)(m+n)$
Start with $1, 2, 3, \ldots$

Sequences (2.4)

Geometric progression : $a, ar, ar^2, ar^3 \ldots$
Aritmetic progression : $a, a+d, a+2d, \ldots$

Summation $\sum_{j=m}^{n} a_j$

Ex) $\sum_{j=0}^{100} \left(\frac{1}{2}\right)^j = \frac{1-\left(\frac{1}{2}\right)^{101}}{\left(\frac{1}{2}\right)} = 2 \cdot \left(1 - \left(\frac{1}{2}\right)^{101}\right)$

$$\sum_{j=1}^{100} j = \frac{100 \cdot (101)}{2} = 5050$$

Two sets A and B have the same cardinality $\Leftrightarrow$ There is a bijection from A to B.

Countable sets have the same cardinality as positive integers.

$\mathbb{Q}$ is countable! $1, \frac{1}{2}, 2, 3, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \ldots$

$\mathbb{R}$ is not countable. Not even the decimals using the digits 6 and 7 are countable.

$x_1 = 0,6666\ldots$
$x_2 = 0,67666\ldots$
$x_3 = 0,76666\ldots$
$\vdots$

$0,767,\ldots$ is not in the list!