# Logistic map

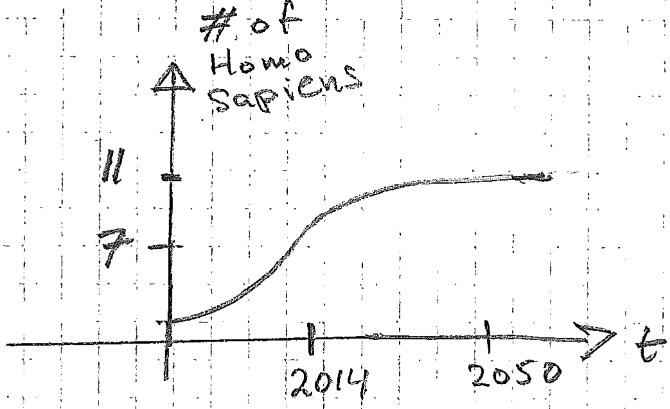
Exponential growth

$$X(0) = 1$$
$$X'(t) = r \cdot X(t)$$
$$X(t) = e^{rt}$$

But not realistic for a large population. Better is

$$X'(t) = r\,X(t) - b\,X(t)^2$$

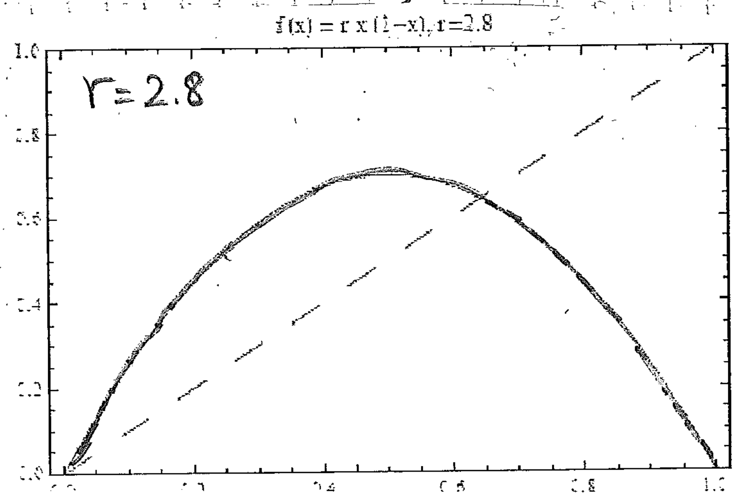
will slow down the growth.




Here $b = r$ and $X_0 \in [0,1]$

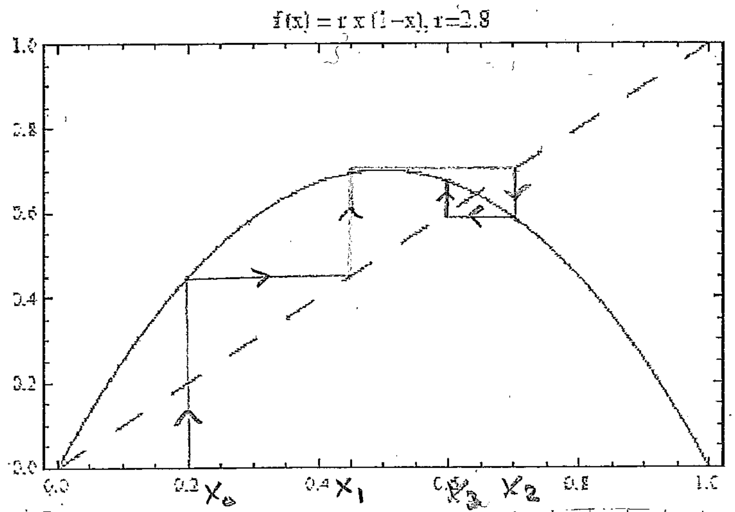
$$X_{n+1} = r\,X_n \cdot (1 - X_n).$$

A non-linear RR, First order and homogeneous.



Graphical illustration for $x_0 = 0.2$

$r = 2.8$




There is a fix point near $x = 0,65$. For fixed point $x^*$

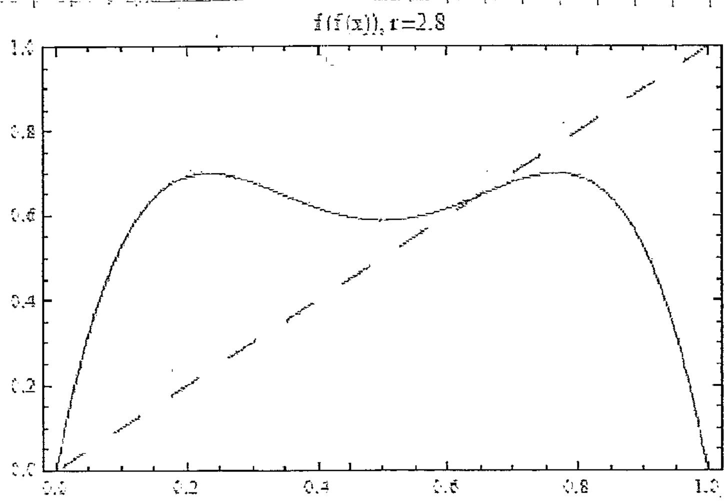
$$x^* = 2.8\, x^* \cdot (1 - x^*)$$

$$x^* = 1 - \frac{1}{2.8}$$

Also interesting to plot is $f(f(x))$.
A polynomial of degree 4.
It tells us what happens after 2 iterations.

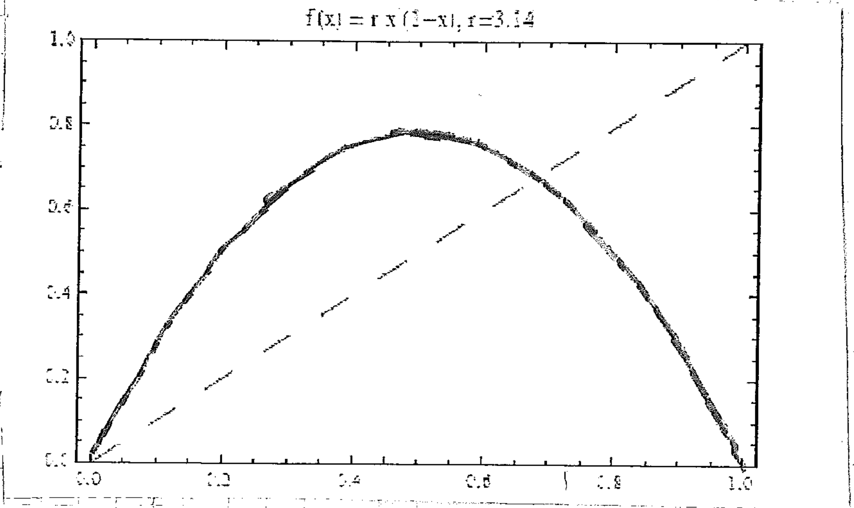
$f(f(x))$

$r = 2.8$



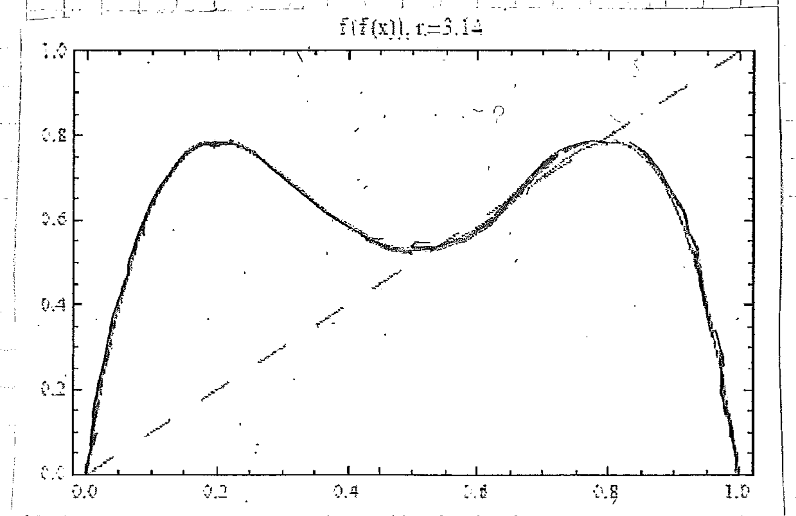
Let us now increase r to 3.14.

r=3.14




The fix point is not attracting any longer. It is unstable.
The difference is seen when we plot $f(f(x))$

$f(f(x))$
$r=3.14$




There are now 3 fix points, the fix point for $f(x)$ and two new. They are part of a 2-cycle.

On the last page you see what happens when r is increased even more.

Around 3.45 the 2-cycle loses its stability and a stable 4-cycle is born. This is called a <u>bifurcation</u>. 

Then $4 \to 8 \to 16 \to 32 \to \ldots$ a process that ends around $r = 3.57$.

A cascade of period doublings. *

For higher values of r we see chaos with some windows of stability.

The Feigenbaum constant appears for many different maps!

*) $$r_k \approx 3.57 - \frac{2.63}{4.67^k} \qquad k = 1, 2, 3, \ldots$$

# Feigenbaum's F=4.669202, r(k)=3.57 – 2.63/F**k