Cellular Automata

1D CA:

today

tomorrow

■ = happy

□ = sad

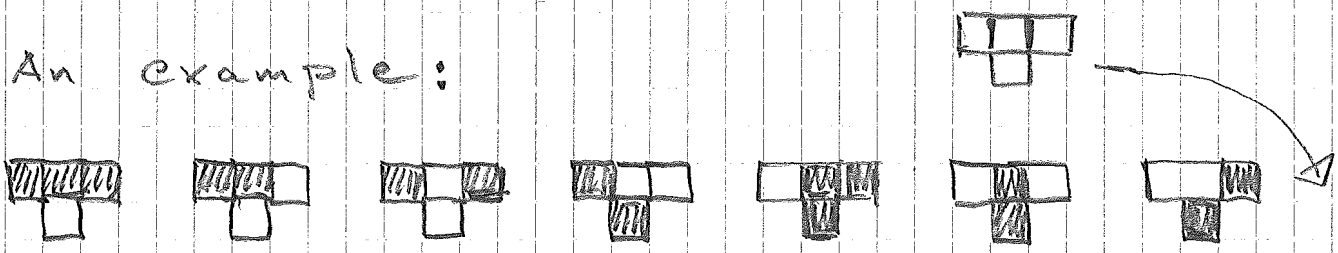
What happens in the future?

You and your nearest-neighbors  can be in $2 \cdot 2 \cdot 2 = 2^3 = 8$ different states

For each such state we have to specify what happens in the middle cell in the next generation (next step). If we do that we have a rule.

How many rules do we have?

An example:

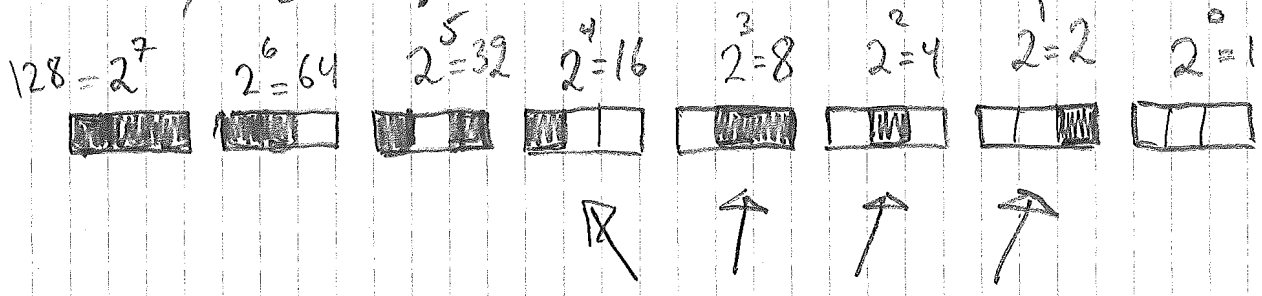


"If middle cell and right cell both are white the middle cell takes the value of the left cell, if they are not both white then the middle cell takes opposite value of left cell".

There are $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 =$
$2^8 = 256$ different rules.
Most of them are uninteresting and predictable. However rule 30 and 110 are fascinating!

The rule given on previous page is called rule 30.

Why 30?



$16 + 8 + 4 + 2 = 30$

Q: How is rule 110 looking like?

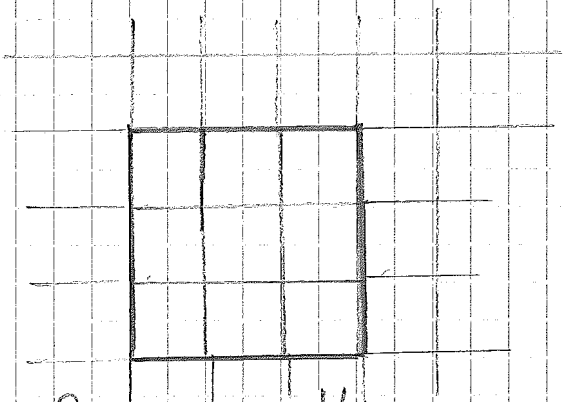
Here we only consider <u>two</u> states for each cell and only <u>nearest - neighbours</u> can influence the value of a cell in next generation (together with the value of the cell itself)

Q: Three states: black, grey, white and  2 neighbours on each side. How many rules?

Answer: $3^{(3^5)} = 3^{243}$ Many rules!

2DCA:

Now 8 neighbors to middle cell



These $1+8=9$ cells can each be in ▨ or □ State so we have $2^9 = 512$ configurations for the 9 cells. For each of these configurations we have to specify what happends in middle in next generation. This gives 2^{512} different rules!

Therefore we consider only rules that depend on the number of neighbors, not their exact location.

Game of life (Conway 1970)

□ → ▨ Birth

▨ → □ Death

▨ → ▨ Survival

As in real life death is due to loneliness (0 or 1 neighbor) or over-population (4 to 8 neighbors).

I am here (!) due to my two parents but in Game of Life birth happens when the middle cell has <u>three</u> living neighbors.

Game of Life is one of $2^{2 \cdot 9} = 2^{18}$ such rules. Since the middle cell is surrounded by 0, 1, 2, 3, 4, 5, 6, 7 or 8 living cells and itself can be alive or dead, $2 \cdot 9 = 18$.

In Mathematica Game of Life is rule number 224. The coding is the following:

N = # neighbours	State of middle cell, M	2N+M	2^{2N+M}	
0	0	0	1	
0	1	1	2	
1	0	2	4	
1	1	3	8	
2	0	4	16	
2	1	5	32	← Give black cell in next generation.
3	0	6	64	←
3	1	7	128	←
4	0	8	256	
4	1	9	512	
⋮	⋮			

128 + 64 + 32 = 224

And in the begining there was a seed ------

On last page I show 100 iterations for rule 746 starting with  . This rule has the same birth condition as Game of Life but the survival condition is different: a living cell survives only if no more than 4 of its 8 neighbours are also alive.

This is mathematics for biology (hopefully). von Neumann and Ulam started it in 1950's.

John Conway discoverd GOL 1970
" Mathematics is the simple bit, it is cats that are complicated."

Stephen Wolfram founded Mathematica and published "A new kind of Science" 2002.

Simple rules can give complex behaviour if repeated many times!

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