Mathematica problems

1. Plot the vector field
$$\bar{A} = (2x - y, 2y - x) \tag{1}$$
See Ex 2.6.13. $\bar{A} \cdot (1, y') = 0$, that is $(1, y')$ is perpendicular to $\bar{A}$. Think of how the solution will look like if $y(1) = 3$. Will it exist for all times?

2. Check the non-linear problem
$$x'(t) = x^2 \tag{2}$$
and $x(0) = 1$. Will Mathematica solve it?

3. Continue with the non-linear problem
$$x'(t) = x^2(t) - t \tag{3}$$
and $x(0) = 1$. How is the output looking like? Conclusion?

4. Plot the vector field
$$\bar{A} = (1, x - y) \tag{4}$$
Solve
$$y'(x) = x - y(x) \tag{5}$$
when $y(-1) = 0$. Plot the solution. Put the two plots together by using Show[plot1,plot2].

5. Solve the matrix problems on "Things you should know....." with Mathematica.

6. Back to problem 2. Solve it with Picard's method. Iterate 6 times. You know the exact solution. Compare with output.

7. Try numerical solution of problem 3. You use NDSolve instead of DSolve. Note that initial and final value of t must be specified.

8. Find the solution to the initial value problem $y''(t) = y(t)$ and $y(0) = 5/4$ and $y'(o) = -3/4$. Plot the solution for $0 \leq t \leq 2$ and determine its minimum value.

9. Consider the system
$$x'(t) = (2 + x)(y - x)$$
$$y'(t) = (4 - x)(y + x)$$

Plot the vector field. Can you see the fix points? Find them by hand and by Mathematica and plot the field around them. Plot also the field for the linearsied system round (0,0). Plot solutions for some start values (0.3,0.3), (-2.2,2.2) and (4.5, 3.7). Do it for $-3 < t < 3$ so you get some of the history and the future!