Mathematics
DFM
Linneaus University

Partial differential equations
4MA103
fall 2012

Some problems on Chapter 5 with help of Mathematica

1. Solve the following three Euler equations for x>0. How do they behave near x=0?

$2x^2y''+3xy'-y=0$ $\quad$ $x^2y''+5xy'+4y=0$ $\quad$ $x^2y''+xy'+y=0$

2. Consider the Laguerre equation

$$xy'' + (1-x)y' + \lambda y = 0.$$

Here λ is a real number. This ODE appears when solving the Schrödinger equation for the H-atom. Is $x = 0$ a regular singular point? Try to find a regular solution, that is it is defined at $x = 0$. Show that when $\lambda = m$, a positive integer, this solution reduces to a polynomial. Find them for $n = 1, 2, 3$. Properly normalized, these polynomials are known as the Laguerre polymomials, $L_m(x)$.

3. Plot the Bessel functions $J_0(x)$, $Y_0(x)$, $J_1(x)$ and $Y_1(x)$ in the interval $0 < x \leq 14$. The Mathematica commands for the functions are BesselJ[m,x] and BesselY[m,x] where m=0 or 1. The Bessel differential equation appears for example when solving the wave equation for a circular membrane,

$$x^2y'' + xy' + (x^2 - m^2)y = 0.$$

The zeros of $J_m(x)$ play an important role for the solution (see 11.4) and Mathematica has the command BesselJZero[m, k] for the k:th zero of $J_m(x)$. Get the first zeros for m=0 and 1.