Mathematics
DFM
Linneaus University

Partial differential equations
4MA103
fall 2012

Some more problems on lectures 1-8 with help of Mathematica

1. It is possible to solve first order PDE with Mathematica. See pdf file at webpage for the course. Note how to write partial derivatives, like D[u[x,y],x] for partial derivative of u(x,y) with respect to x.Solve

$$2\frac{\partial u}{\partial x_1} - \frac{\partial u}{\partial x_2} = 0 \tag{1}$$

when $u(x_1, 0) = e^{-x_1^2}$.

2. If a square has side length π the eigenvalue problem $(\Delta + k^2)\psi = 0$ has with Dirichlet boundary conditions(that means $\psi = 0$ on the boundary) the following solution

$$k^2 = m^2 + n^2 \tag{2}$$

$$\psi_{mn}(x,y) = A_{mn} sin(mx) sin(ny) \tag{3}$$

Here m and n are positive integers and A_{mn} a constant that is normally fixed by the normalization condition

$$\int\int_D \psi_{mn}^2 dxdy = 1 \tag{4}$$

The domain of integration D is the square $0 \leq x \leq \pi, 0 \leq y \leq \pi$. Plot like in lecture 3

$$u(x,y,v) = cos(v)\psi_{mn}(x,y) + sin(v)\psi_{nm}(x,y) \tag{5}$$

for some $m \neq n$ and v. The parameter v lies between 0 and π.

3. Consider the ODE

$$4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0.$$

Is $x = 0$ a regular singular point? Solve the problem with Mathematica (use DSolve) and plot the regular solution $y_1(x)$ for $x > 0$. Do it also with paper and pen and use Frobenius method and compare. The second solution is hard to find so try instead ansatz $y_2(x) = y_1(x)w(x)$. Put it into the ODE and solve the new ODE for w.

4. This week we will start with eigenvalue problems for differential operators. Start with

$$-y''(x) = \lambda y(x).$$

and boundary condition $y(0) = 0$. We assume here that $\lambda > 0$. The function is now determined up to a multiplication by a constant. Put this constant equal to 1 and plot the function as function of x for some λ values in the region [0.5,10]. Plot also $y(\pi)$ as function of λ in the whole interval. If the second boundary condition is $y(\pi) = 0$ which λ values are now allowed in the region? These are the eigenvalues to this eiegenvalue problem with boundary condition $y(\pi) = y(0) = 0$