

### 11.3 Non-homogeneous B.V.P.

Due to heat sources,  
external forces, charges, .....

$$(*) \quad L y(x) = \mu r(x) y(x) + \underline{\underline{f(x)}}$$

$\mu$  a constant, often zero.

Same separated homogeneous  
boundary conditions as before.

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \dots$$

$$\phi_1(x), \phi_2(x), \phi_3(x), \dots$$

Solution to  $L y(x) = \lambda r(x) y(x)$

$$\text{Ansatz: } \phi(x) = \sum_{n=1}^{\infty} b_n \phi_n(x)$$

for the solution to (\*)

$$f(x) = \frac{f(x) \cdot r(x)}{r(x)} = r(x) \sum_{n=1}^{\infty} c_n \phi_n(x)$$

$$c_n = (f, \phi_n)$$

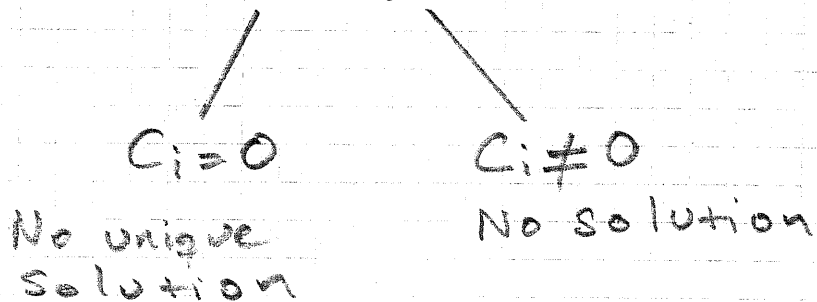
All this into  $(x)$  gives

$$\sum_{n=1}^{\infty} [(\lambda_n - \mu) b_n - c_n] \phi_n(x) = 0$$

$$b_n = \frac{c_n}{\lambda_n - \mu}$$

If  $\mu = 0$  and  $\lambda = 0$  is not an eigenvalue then there is a unique solution

If  $\mu = 0$  and  $\lambda := 0$  is an eigenvalue



Do 11.3.1

$$-y''(x) = 2y + x$$

$$y(0) = 0$$

$$y(1) = 0$$