11.3 Non-homogenous B.V.P.

Due to heat sources, external forces, charges .....

(*) $Ly(x) = \mu r(x) y(x) + \underline{\underline{f(x)}}$

μ a constant, often zero.

Same separated homogenous boundary conditions as before.

$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \ldots$

$\phi_1(x), \phi_2(x), \phi_3(x) \ldots$

solution to $Ly(x) = \lambda r(x) y(x)$

Ansatz: $\phi(x) = \sum_{n=1}^{\infty} b_n \phi_n(x)$

for the solution to (*)

$$f(x) = \frac{f(x) \cdot r(x)}{r(x)} = r(x) \sum_{n=1}^{\infty} c_n \phi_n(x)$$

$$c_n = (f, \phi_n)$$

All this into (*) gives

$$\sum_{n=1}^{\infty} \left[(\lambda_n - \mu) b_n - c_n \right] \phi_n(x) = 0$$

$$b_n = \frac{c_n}{\lambda_n - \mu}.$$

If $\mu = 0$ and $\lambda = 0$ is not an eigenvalue then there is a unique solution

If $\mu = 0$ and $\lambda_i = 0$ is an eigenvalue

$c_i = 0$
No unique solution

$c_i \neq 0$
No solution

Do 11.3.1 $\quad -y''(x) = 2y + x$

$$y(0) = 0$$
$$y(1) = 0$$