Mathematics | Partial differential equations
DFM | 4ma103
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Home exam

T1. Find eigenvalues and eigenfunctions to the boundary value problem

$$y'' + y' + \lambda y = 0, \qquad y(0) = 0,\ y(2) = 0 \tag{1}$$

Transfer the differential equation to self-adjoint form. How is the orthogonality relation looking like?

T2. Determine the solution to the differential equation

$$x''(t) + \frac{3}{2t}x'(t) + x(t) = 0 \tag{2}$$

, $t > 0$, which is bounded when $t \to 0^+$.

T3. Solve the Laplace equation for a cylinder. (This could be the heat equation for a stationary state). You can assume radial symmetry, i.e. no dependence on the angle. Thus

$$\begin{aligned} \Delta u(r,z) &= 0 \\ u(2,z) &= 0, \qquad 0 < z < 4 \\ u(r,0) &= 0,\ u(r,4) = 2, \qquad 0 < r < 2. \end{aligned}$$

The answer shall be given as a series. You do not have to calculate the coefficients in the expansion but you have to explain how to determine them. A spectral theorem can be assumed. Hint: Use cylinder coordinates. The solution can not be periodic in the z-coordinate. The solution must be bounded at $r = 0$.

N1. Calculate the coefficients in problem T3 and plot the sum with 10,20,30 terms.

N2. There are two eigenvalues below 4 to the SL-problem

$$-y''(x) - \frac{5}{1+x^2}y(x) = \lambda y \tag{3}$$

,$y(\pi) = y(2\pi) = 0$. Find them and plot the corresponding eigenfunctions. You can put $y'(0) = 1$ an run the initial-value problem for different values of λ and see when you fit the BC at the right endpoint.