

Problems 10/11

1) Wave equation $\frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 U}{\partial x^2} = 0$

Solve it by change of variables

$$\xi = x - ct$$

$$\eta = x + ct$$

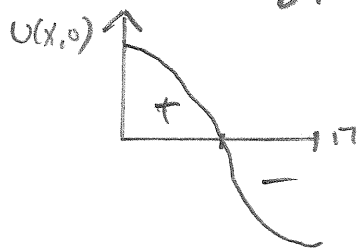
Determine the 2 unknown functions

to IV $U(x, 0) = U_0(x) = \text{graph} \rightarrow x$
 $\frac{\partial U}{\partial t}(x, 0) = 0$

2) Heat equation $\frac{\partial U}{\partial t} - \frac{\partial^2 U}{\partial x^2} = 0$

IV: $U(x, 0) = \cos^3 x \quad 0 \leq x \leq \pi$

BC: $\frac{\partial U}{\partial x}(0, t) = \frac{\partial U}{\partial x}(\pi, t) = 0 \quad t > 0$



3) Laplace equation $\Delta U = 0 \quad |\bar{r}| < 1$
 $U = f(\varphi) \quad |F| = 1$



solve it!

$$U(r, \varphi) = P(r) \Phi(\varphi)$$

$$\Delta = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2}$$

Problems 17/11

1) Can you express

$$\frac{x}{2} + \frac{2x^2}{4} + \frac{3x^3}{8} + \dots = \sum_{n=1}^{\infty} \frac{n}{2^n} x^n$$

with help of elementary functions?

Hint: $\frac{1}{1-x} = 1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \dots$

2) Solve $y''(x) = -y(x)$
with the ansatz $y(x) = \sum_{n=0}^{\infty} a_n x^n$

3) Show that the ansatz $y(x) = \sum_{n=0}^{\infty} a_n x^n$
doesn't work for the ODE

$$2x^2 y''(x) + 3xy' - (1+x)y = 0$$

Here $x=0$ is a regular singular point.

4) $\frac{\partial U}{\partial x_1} + 2\frac{\partial U}{\partial x_2} = 1$

$U(0, x_2) = x_2^2$. What is $U(1, 5)$?
(see my paper on first-order PDE)

5) Which is the third lowest eigenvalue

for $-\Delta \psi = k^2 \psi$

with negative parity? The problem is
for a 2D box with side length π , $\psi=0$
(see my paper on 2D-box) at boundary