PDE problems 24/11

1. We showed yesterday that
$$4x^2y''(x) - 8x^2y'(x) + (4x^2+1)y(x) = 0$$
has solution $y_1(x) = \sqrt{x}\, e^x$
Since $r = 1/2$ is a double root the second solution looks like
$$y_2(x) = y_1(x)\ln x + \sqrt{x}\sum_{n=1}^{\infty} b_n x^n.$$
Try instead $y_2(x) = y_1(x)\, w(x)$
Solve ODE for $w(x)$.

2) $$3x^2y''(x) + 2xy'(x) + x^2y = 0\,, \quad x>0$$
again! $\quad r=0 \quad a_0 = 1,\ a_2 = -1/10,\ a_4 = \frac{1}{440}$
$\quad r = 1/3 \quad a_0 = 1,\ a_2 = -1/14\ \ a_4 = \frac{1}{728}$

The coefficient in front of x^{r+1} is $a_1(r+1)(3r+2)$. Zero if $r_1 = -1$, $r_2 = -2/3$. New solutions?

3) (5.6.13) $$xy'' + (1-x)y' + \lambda y = 0\,, \quad x>0$$
is the Laguerre equation.
Show that $x=0$ is a regular singular point. Determine one solution.
Show that when constant $\lambda = m$ (a positive integer) this solution reduces to a polynomial. When properly normalized this polynomial is known as the Laguerre polynomial $L_m(x)$.