Problem 1. Om man komprimerar en gas, ökar dess temperatur. Varför? When a gas is compressed, its temperature goes up. Why?

a) Give here your reasoning from macroscopic thermodynamics. Ge här en förklaring utifrån makroskopisk termodynamik (1p)

Lösning: 1

Compression means that mechanical work $W = \int p \mathrm{d}V$ is done on the gas, so that its internal energy increases. This means that also the temperature goes up.

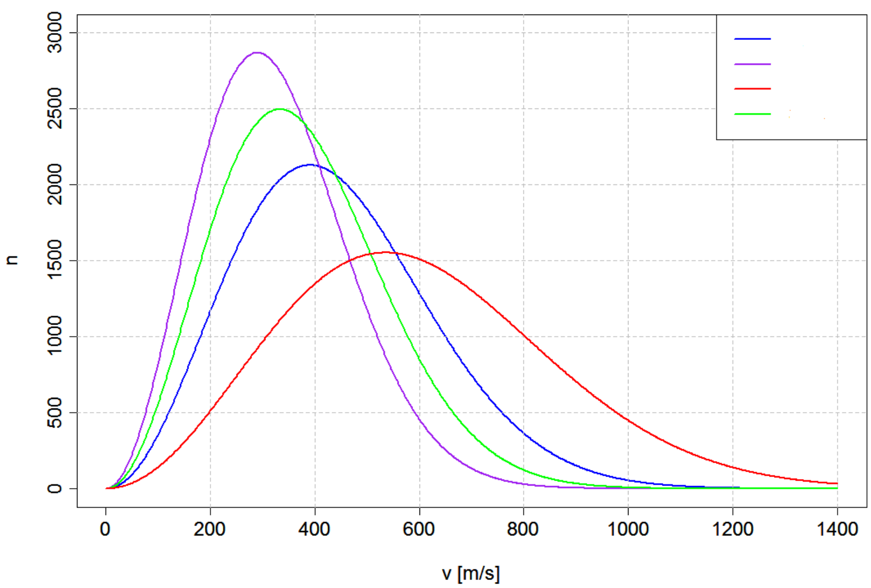
b) Give here an explanation from microscopic physics. Ge här en förklaring utifrån mikroskopisk fysik. (1p)

Lösning: 1

In a microscopic description of compression, molecules bouncing against an inward-moving wall of the container are reflected with a slightly higher speed. If the compression is not small, this will happen many times, so that the increase in kinetic energy is appreciable. This increase of random velocities is an increase in temperature.

Problem 2. Figuren nedan visar antalen gasmolekyler med farten v hos fyra olika gaser (butan C_4H_{10}, syrgas O_2, koldioxid CO_2 och ammonia NH_3) vid en viss given temperatur.

The figure below shows the numbers of molecules with speed v of four different gases (butane C_4H_{10}, oxygen O_2, carbon dioxide CO_2 and ammonia NH_3) at some temperature.



a) Vilken kurva hör till vilken gas? Svaret måste motiveras. Which curve belongs to which gas? Explain your answer. (1p)

Lösning: 1

The average kinetic energy is determined by the temperature ($\frac{3}{2}k_BT$), the same for all four gases. This means that the lightest molecules NH_3 ($m = 17$) are fastest, followed by oxygen O_2 ($m = 32$), carbon dioxide CO_2 ($m = 44$), and lastly heavy butane C_4H_{10} ($m = 58$).

b) Vilken är temperaturen? What is the temperature? (1p)

Lösning: 1

The probability distribution peaks at $v_p = \sqrt{\frac{2kT}{m}}$; the most accurate estimate can be found from the NH_3 curve which peaks around 533 m/s: $T = \frac{mv_p^2}{2k} = \frac{17 \times 1.66 \cdot 10^{-27} \times 533^2}{2 \times 1.38 \cdot 10^{-23}} = 290$ K. This value can be checked with v_p from the other curves.

Problem 3. Consider an amount of water at a pressure of 3.0 bar and a volume of 200 cm^3 at a temperature of 100°C (state A). Consider two different ways of going to state B with a volume of 600 cm^3 and the same temperature. One way is along the isotherm. The other way is an isobaric expansion to a state C followed by an isochore to state B.

a) Calculate the change in internal energy, the heat exchange with the environment and the work on the gas for the three changes A → B, A → C, and C → B . Give answers in joules. (3p)

Lösning: 3

Start by drawing a $p-V$ diagram according to ideal-gas-law behaviour, from which the ΔW can be easily computed:

- $\Delta W_{\text{CB}} = 0$ (no change in volume)
- $\Delta W_{\text{AC}} = -p \times \Delta V = -3 \cdot 10^5 \times 400 \cdot 10^{-6} = -120$ J;
- $\Delta W_{\text{AB}} = -\int_{\text{A}}^{\text{B}} p \text{ d}V = -\int_{200}^{600} \frac{60 \text{ J}}{V} \text{ d}V = -60 \text{ J} \times \ln V|_{200}^{600} = -60 \text{ J} \times \ln 3 = -65.9$ J. It can be seen that this is a reasonable answer: the area below the isotherm from A to B is slightly larger than half of the rectangle below the isobar AC (draw the diagonal).

There is no difference in internal energy between states A and B, because argon behaves as an ideal gas, for which the internal energy does not depend on density at all. Conservation of energy tells us that the amount of heat exchanged along the isotherm $\Delta Q_{\text{AB}} = -\Delta W_{\text{AB}} = 65.9$ J.

In order to calculate quantities of heat and changes in internal energies with respect to state C, we need to know the amount of gas and the temperature in C. The ideal gas law tells us that $T_C = 3\ T_{AB} = 3 \times 373 = 1119$ K. The amount of substance is $n = \frac{pV}{RT} = \frac{60}{8.314 \times 373} = 0.0193$ mole.

The heat released when cooling down from C to B is $\Delta Q_{\text{CB}} = nC_p\Delta T = \frac{60}{R \times 373} \times \frac{3}{2}R \times -2 \cdot 373 = -180$ J. That is also equal to the change in internal energy because $\Delta W_{\text{CB}} = 0$.

The heat absorbed by the gas when heating up from A to C is $\Delta Q_{\text{AC}} = nC_v\Delta T = \frac{60}{R \times 373} \times \frac{5}{2}R \times 2 \cdot 373 = 300$ J. The change in internal energy in this process is $\Delta Q_{\text{AC}} - \Delta W_{\text{AC}} = 300 - 120 = 180$ J.

b) Derive an equation that connects all ΔQ and all ΔW in this problem. Show that the equation computes for the numerical values you found in *a*). (1p)

Lösning: 1

For any substance, the internal energy is a state function - the difference in internal energy between A and B does not depend on the path taken:

$$\Delta U_{\text{AB}} = \Delta U_{\text{AC}} + \Delta U_{\text{CB}}.$$

The first law of thermodynamics gives:

$$\Delta Q_{\text{AB}} + \Delta W_{\text{AB}} = \Delta Q_{\text{AC}} + \Delta W_{\text{AC}} + \Delta Q_{\text{CB}} + \Delta W_{\text{CB}}.$$

Collecting all ΔQ on the left and all ΔW on the right, we get

$$\Delta Q_{\text{AB}} - \Delta Q_{\text{AC}} - \Delta Q_{\text{CB}} = \Delta W_{\text{AC}} + \Delta W_{\text{CB}} - \Delta W_{\text{AB}}.$$

This expression is general, not just for an ideal gas. In our case

$$65.9 - 300 - (-180) = -120 + 0 - (-65.9).$$