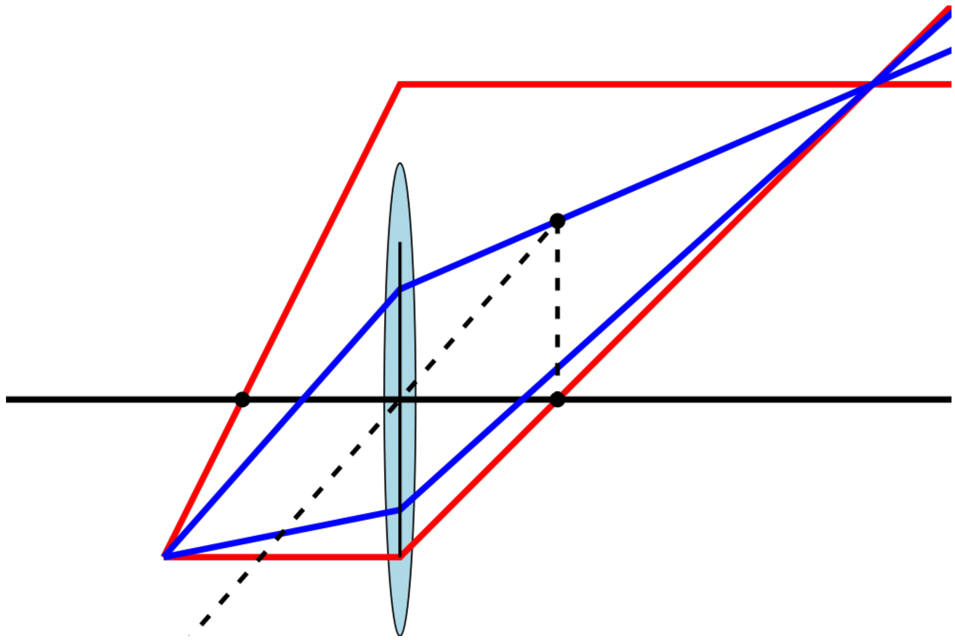
Problem 1. The figure below shows two light rays from a source. The rays are refracted by a lens.



a) Is this a positive or a negative lens? Explain. (1p)

Lösning: 1

Positive - it makes a diverging wavefront converge.

b) Determine by construction the focal points of this lens. Explain your lines. (2p)

Lösning: 2

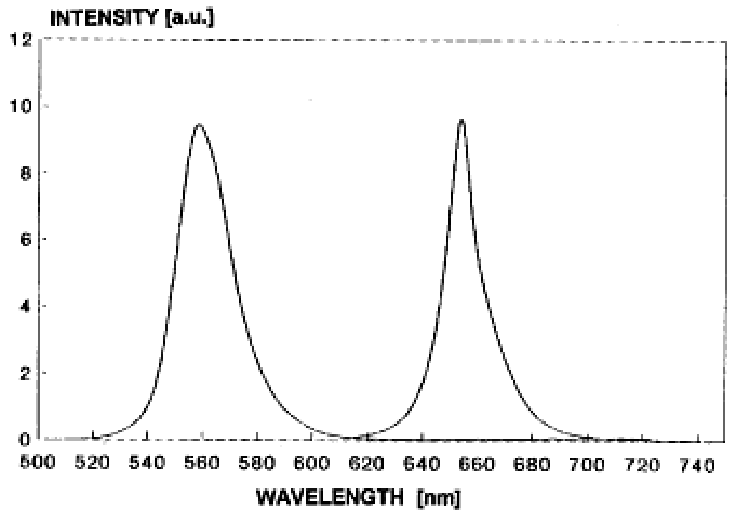
The most straightforward way is to draw the two red lines connecting source and image, parallel to the optical axis on one side of the lens; they pass though its focal points on its other side.

c) There is a second way of determining the focal points by construction. Describe those lines. (1p)

Lösning: 1

The dashed line throught the center of the lens is parallel to one of the rays from the source, so on the other side of the lens it intersects with that ray in the focal plane for sources at infinity; a line perpendicular to the optical axis gives one of the focal points of the lens. Four such lines can be drawn: two give the focal plane on the right, two give the focal plane on the left.

Problem 2. The figure below shows the spectra of two light-emitting diodes.



a) What are their colours? What are their coherence lengths? (1p)

Lösning: The LED at 560 nm is green, the one at 650 nm is red. The red one has a FWHM spectrum of 10 nm, which gives a coherence time of 65 periods, so that $\tau_c = 65 \times \lambda/c$ and $\ell_c = \tau_c c = 65\lambda = 42\mu\text{m}$. Similarly, the green LED has a bandwidth of 20 nm and a coherence length of 28 wavelengths or 18 micrometer. $\boxed{1}$

b) We want to modulate the intensity of the longer wavelength LED to transmit a signal through a 1 km optical fiber. The fiber's index of refraction is given as $n(\lambda) = A + B/\lambda^2$ with $A = 1.466$ and $B = 4.153 \cdot 10^{-15}$ m^2.
What is the phase velocity? (1p)

Lösning: $\boxed{1}$
At 650 nm the refractive index is $n = 1.466 + \frac{4.153 \cdot 10^{-15}}{(0.65 \cdot 10^{-6})^2} = 1.466 + \frac{0.004153}{0.65^2} = 1.476; v_\phi = c/n = 2.031 \cdot 10^8$ m/s.

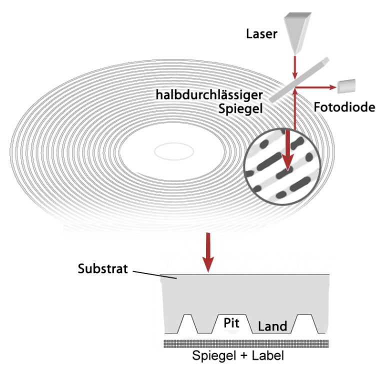
c) How large is the group velocity? (1p)

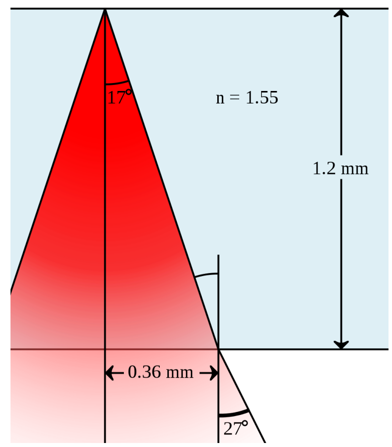
Lösning: $\boxed{1}$
From Hecht problem 7.27, $v_g = \frac{c}{n} + \frac{\lambda c}{n^2}\frac{\mathrm{d}n}{\mathrm{d}\lambda} = v_\phi(1 + \frac{\lambda}{n}\frac{\mathrm{d}n}{\mathrm{d}\lambda}) = v_\phi(1 + \frac{\lambda}{n}\frac{-2B}{\lambda^3}) = v_\phi(1 - \frac{2B}{n\lambda^2}) = v_\phi(1 - \frac{2 \times 0.004153}{1.476 \times 0.65^2}) = v_\phi(0.987) = 2.005 \cdot 10^8$ m/s.

d) What is the highest modulation frequency that can be used? (1p)

Lösning: $\boxed{1}$
The spectral dispersion gives the difference in transit time for 645 and 655 nm light. The difference in refractive index is $\Delta\lambda \times \frac{\mathrm{d}n}{\mathrm{d}\lambda} = \Delta\lambda \times \frac{2B}{\lambda^3} = \frac{\Delta\lambda}{\lambda} \times 2 \times \frac{B}{\lambda^2} = \frac{1}{65} \times 2 \times 0.010 = 3 \cdot 10^{-4}$, or which means a relative difference of about $2 \cdot 10^{-4}$. The transit time is about $\Delta\ell/v_g = \frac{10^3}{2 \cdot 10^8} = 5 \cdot 10^{-6}$ sec. The difference for 645 and 655 nm will be about a nanosecond, which means that the maximum frequency will be about 0.5 GHz.

Problem 3. The figure below illustrates the readout system of a Compact-Disc player (upside down and not to scale). The label 'halbdurchlässiger Spiegel' is German for semitransparent mirror. The laser has a wavelength of 780 nm. The laser optics has a numerical aperture of 0.45. The CD consists of a transparent polycarbonate substrate of 1.2 mm thickness; its index of refraction is 1.55. The information is encoded in a highly reflective metallized layer with shallow pits. The pits are 0.67μm wide and of varying length.





a) The diode detects less light when the laser spot is focused on a pit because of destructive interference when half the light is reflected from 'land' and half from 'pit'. How large should the height difference between pits and land be to achieve the largest difference in intensity on the diode? (1p)

Lösning: Destructive interference occurs at a path difference of half a wavelength in the material, so at $d = \frac{\lambda}{4n} = \frac{780}{4\times1.55} = 125$ nm; but a smaller difference works better for the servo tracking system. $\boxed{1}$

b) Where does the energy of the incident laser beam go when it is focused on a pit? (1p)

Lösning: $\boxed{1}$

The incident energy is reflected in diffracted beams. The situation is similar to diffaction by a narrow transparent obstacle that gives the light a $\lambda/2$ retardation.

c) The size of the focus is limited by diffraction. In order to minimize cross talk between neighbouring tracks, they should be separated by the first minimum of diffraction. What would be a suitable distance between tracks? (1p)

Lösning: $\boxed{1}$

The Airy disc has its first minimum at 1.22λ times the f-number, or for optics for finite conjugates at $\frac{1.22\lambda}{2\times NA} = \frac{1.22\times0.78}{2\times0.45} = 1.06$ μm. However, in practice the diffraction limit is not attained and positioning is not infinitely accurate. That is why the CD specifications prescribe a track pitch of 1.6 μm.

c) The focus is produced by a convergent beam coming from below, so that readout is insensitive to specks of dust on the surface. How large is the diameter of the laser beam at the lower surface? Make a clear sketch beside the figure above. (1p)

Lösning: The Numerical Aperture gives $\theta_{max} = \arcsin 0.45 = 27°$. Applying Snell's law gives the angle of the light cone inside the polycarbonate. The radius is $1.2 \times \tan(\arcsin \frac{0.45}{1.55}) = 1.2 \times \tan 16.9° = 0.36$ mm; the diameter is 0.72 mm (often rounded off to 0.8 mm). $\boxed{1}$