Problem 1. Solargraphy is a method of recording the ecliptic (the path of the Sun over the sky) using a pinhole camera. The Sun's apparent diameter is about $\frac{1}{2}^\circ$. In the method of the Finnish artist Tarja Trygg, the camera is a standard black-plastic film canister (diameter is 30 mm), with ordinary photographic paper as the sensor.

a) Estimate from geometrical optics the size of the largest pinhole that can resolve objects of the size of the Sun. (1p)

Lösning: 1

Half a degree is about 0.01 radians; assuming the size of the pinhole is negligible, geometrical optics gives an image size of 0.3 mm; the hole should not be larger if this is the required resolution.

b) You decide to use a small pinhole to try to make sharper images and you choose 0.01 mm. Using all optics taught in this course, what is the size of the image of the Sun on the photographic paper? Does this resolve the Sun? (1p)

Lösning: 1

Diffraction gives an angular width of about $\frac{\lambda}{d} = \frac{0.5\ \mu m}{10\ \mu m} = 0.05$ radians; this does not resolve images of the size of the sun.

Problem 2. Consider a multi-mode stepped-index optical fiber with an attenuation of 0.3 dB/km. The refractive index of the core is 1.500, the cladding has an index of 1.485.

a) What is the critical angle inside the fiber? (1p)

Lösning: 1

$\theta_c = \arcsin \frac{1.485}{1.500} = \arcsin \frac{1.500-0.015}{1.500} = \arcsin(1-0.01) = \arcsin 0.99 \approx 82^\circ$.

b) How large is the numerical aperture of this fiber? (1p)

Lösning: 1

N.A. $= \sqrt{n_{core}^2 - n_{clad}^2} \approx \sqrt{0.015 \times 3} \approx \sqrt{0.04} = 0.2$.

c) How large is the mode dispersion (in ns/km) of this fiber? (1p)

Lösning: 1

The minimum time for 1 kilometer of fiber is $Ln_{core}/c = 10^3 \times 1.5/3.10^8 = 0.5 \times 10^{-5}$ s. The path for a meridional ray with the critical angle is longer by $1/\cos\theta_t = 1/\sin\theta_c = 1/0.99 = 1$ %. So the intermodal delay of this stepped index fiber is 50 ns/km.

c) After what length of this fiber has the intensity dropped to 1%? (1p)

Lösning: 1

1 % is 20 dB attenuation, which occurs after $20/0.3 = 67$ km.

Problem 3. Turpentine has a refractive index of 1.472 and a rotatory power of $-0.37°\text{mm}^{-1}$ (at 10°C, $\lambda = 589.3$ nm).

a) A 10 mm wide cuvette with turpentine is placed between crossed polarizers. Disregarding reflection and absorption, how much light is transmitted through the setup? (1p)

Lösning: 1

The rotation of sodium D light by 10 mm of turpentine is 3.7°; for the second polarizer, Malus' law gives a transmission of $\cos^2(90° - 3.7°) = \sin^2 3.7° = 0.065^2 =$ 0.4 %; when starting with unpolarized light, the total transmission is 0.2 %.

b) How large is the absolute value of the difference between the refractive indices for left-handed and right-handed circularly polarized light in turpentine? (1p)

Lösning: 1

After travelling 10 cm, the rotation is $370° \approx 2\pi$. The LCP and the RCP then have accumulated a phase difference of 4π, or an optical path difference of 2λ. We get $|n_{\mathcal{L}} - n_{\mathcal{R}}| = \Delta n = \frac{2\lambda_0}{d} = \frac{2\times 0.589\times 10^{-6}}{0.10} = 1.2 \times 10^{-6}$.

Problem 4. *a*) Large magnifying glasses for reading have much smaller magnifications than jewellers' loupes. Explain why. (1p)

Lösning: 1

Large magnifications are achieved by short focal lengths ($M = 10\times$ implies $f = 2.5$ cm); this is just geometrically impossible for a lens with a large diameter. Also, lenses with a large diameter must have large radii of curvature of the refracting surfaces, which gives long focal lengths and low magnifications.

b) What use is such a large magnifying glass for an elderly Sherlock Holmes when its nominal magnification is less than 2? (1p)

Lösning: 1

For a person with the eyesight that is normal for young people, a nominal magnification of 1.25 would hardly matter. It means that the magnifier has a focal length of 20 cm. But if Sherlock's near point moved to 50 cm, that magnifier would help him investigate clues at a size 2.5 times as large as with his unaided eye.
