

Duane-Hunt relation and determination of Planck's constant

Objects of the experiment

- To determine the limit wavelength λ_{\min} of the bremsstrahlung continuum as a function of the high voltage U of the x-ray tube.
- To confirm the Duane-Hunt relation.
- To determine Planck's constant.

Principles

The bremsstrahlung continuum in the emission spectrum of an x-ray tube is characterized by the limit wavelength λ_{\min} (see Fig. 1), which becomes smaller as the tube high voltage increases (see experiment P6.3.3.2). In 1915, the American physicists *William Duane* and *Franklin L. Hunt* discovered an inverse proportionality between the limit wavelength and the tube high voltage:

$$\lambda_{\min} \sim \frac{1}{U} \quad (I).$$

This Duane-Hunt relationship can be sufficiently explained by examining some basic quantum mechanical considerations: As the wavelength λ and the frequency ν for any electromagnetic radiation are related in the manner

$$\lambda = \frac{c}{\nu} \quad (II)$$

$c = 2.9979 \cdot 10^8 \text{ m s}^{-1}$: velocity of light

the minimum wavelength λ_{\min} corresponds to a maximum frequency ν_{\max} respectively a maximum energy

$$E_{\max} = h \cdot \nu_{\max} \quad (III)$$

h : Planck's constant

of the emitted x-ray quanta. However, an x-ray quantum attains maximum energy at precisely the moment in which it acquires the total kinetic energy

$$E = e \cdot U \quad (IV)$$

$e = 1.6022 \cdot 10^{-19} \text{ A s}$: elementary charge

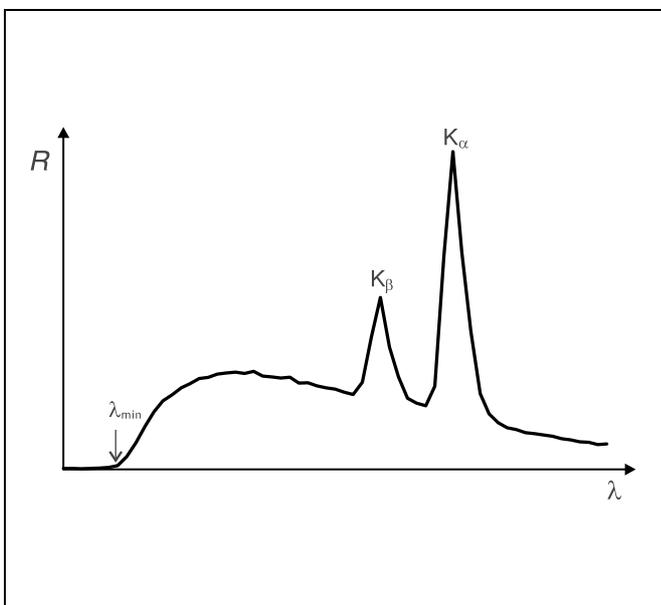
of an electrode decelerated in the anode. It thus follows that

$$\nu_{\max} = \frac{e}{h} \cdot U \quad (V)$$

respectively

$$\lambda_{\min} = \frac{h \cdot c}{e} \cdot \frac{1}{U} \quad (VI)$$

Fig. 1 Emission spectrum of an x-ray tube with the limit wavelength λ_{\min} of the bremsstrahlung continuum and the characteristic K_{α} and K_{β} lines.



Apparatus

- 1 X-ray apparatus 554 811
 - 1 End-window counter
for α , β , γ and x-ray radiation 559 01
- additionally required:
- 1 PC with Windows 95/98/NT

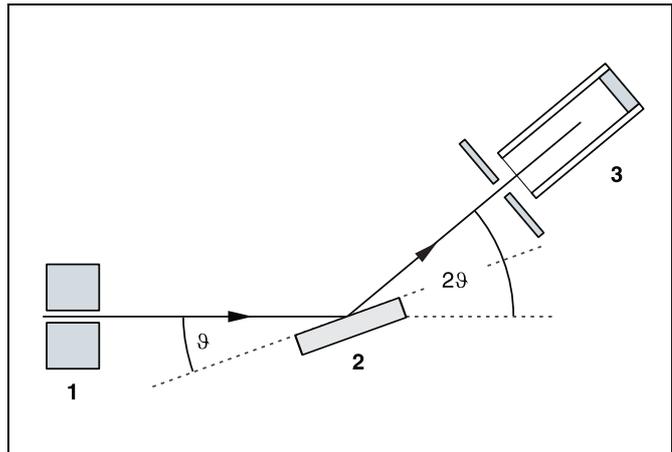


Fig. 2 Schematic diagram of diffraction of x-rays at a monocrystal and 2θ coupling between counter-tube angle and scattering angle (glancing angle)
1 collimator, 2 monocrystal, 3 counter tube

Equation (VI) corresponds to Duane and Hunt's law. The proportionality factor

$$A = \frac{h \cdot c}{e} \quad \text{(VII)}$$

can be used to determine Planck's constant h when the quantities c and e are known.

A goniometer with NaCl crystal and a Geiger-Müller counter tube in the Bragg configuration together comprise the spectrometer in this experiment. The crystal and counter tube are pivoted with respect to the incident x-ray beam in 2θ coupling (cf. Fig. 2).

In accordance with Bragg's law of reflection, the scattering angle ϑ in the first order of diffraction corresponds to the wavelength

$$\lambda = 2 \cdot d \cdot \sin \vartheta \quad \text{(VIII)}$$

$d = 282.01$ pm: lattice plane spacing of NaCl

Safety notes

The x-ray apparatus fulfills all regulations governing an x-ray apparatus and fully protected device for instructional use and is type approved for school use in Germany (NW 807/97 Rö).

The built-in protection and screening measures reduce the local dose rate outside of the x-ray apparatus to less than $1 \mu\text{Sv/h}$, a value which is on the order of magnitude of the natural background radiation.

- Before putting the x-ray apparatus into operation inspect it for damage and to make sure that the high voltage is shut off when the sliding doors are opened (see Instruction Sheet for x-ray apparatus).
- Keep the x-ray apparatus secure from access by unauthorized persons.

Do not allow the anode of the x-ray tube Mo to overheat.

- When switching on the x-ray apparatus, check to make sure that the ventilator in the tube chamber is turning.

The goniometer is positioned solely by electric stepper motors.

- Do not block the target arm and sensor arm of the goniometer and do not use force to move them.

Setup

Setup in Bragg configuration:

Fig. 3 shows some important details of the experiment setup. To set up the experiment, proceed as follows (see also the Instruction Sheet for the x-ray apparatus):

- Mount the collimator in the collimator mount (a) (note the guide groove).
- Attach the goniometer to guide rods (d) so that the distance s_1 between the slit diaphragm of the collimator and the target arm is approx. 5 cm. Connect ribbon cable (c) for controlling the goniometer.
- Remove the protective cap of the end-window counter, place the end-window counter in sensor seat (e) and connect the counter tube cable to the socket marked GM TUBE.
- By moving the sensor holder (b), set the distance s_2 between the target arm and the slit diaphragm of the sensor receptor to approx. 6 cm.
- Mount the target holder (f) with target stage.
- Loosen knurled screw (g), place the NaCl crystal flat on the target stage, carefully raise the target stage with crystal all

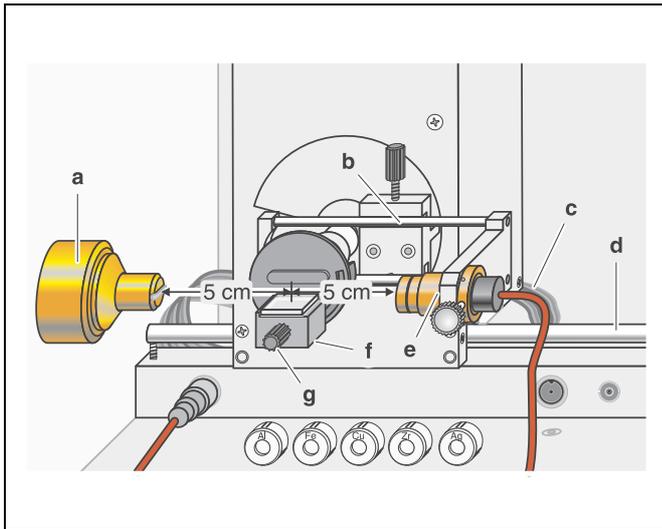


Fig. 3 Experiment setup in Bragg configuration

the way to the stop and gently tighten the knurled screw (prevent skewing of the crystal by applying a slight pressure).

- If necessary, adjust the mechanical zero position of the goniometer (see Instruction Sheet for x-ray apparatus).

Notes:

NaCl crystals are hygroscopic and extremely fragile. Store the crystals in a dry place; avoid mechanical stresses on the crystal; handle the crystal by the short faces only.

If the counting rate is too low, you can reduce the distance s_2 between the target and the sensor somewhat. However, the distance should not be too small, as otherwise the angular resolution of the goniometer is no longer sufficient.

Preparing the PC-based measurement:

- Connect the RS-232 output and the serial interface on your PC (usually COM1 or COM2) using the 9-pin V.24 cable (supplied with x-ray apparatus).
- If necessary, install the software "X-ray Apparatus" under Windows 95/98/NT (see Instruction Sheet for x-ray apparatus) and select the desired language.

Carrying out the experiment

- Start the software "X-ray Apparatus", check to make sure that the apparatus is connected correctly, and clear any existing measurement data using the button or the F4 key.
- Set the tube high voltage $U = 22$ kV, the emission current $I = 1.00$ mA, the measuring time per angular step $\Delta t = 30$ s and the angular step width $\Delta\beta = 0.1^\circ$.
- Press the COUPLED key to activate 2θ coupling of target and sensor and set the lower limit of the target angle to 5.2° and the upper limit to 6.2° .
- Start measurement and data transfer to the PC by pressing the SCAN key.

Tab. 1: Recommended parameters for recording the measurement series

$\frac{U}{\text{kV}}$	$\frac{I}{\text{mA}}$	$\frac{\Delta t}{\text{s}}$	$\frac{\beta_{\min}}{\text{grd}}$	$\frac{\beta_{\max}}{\text{grd}}$	$\frac{\Delta\beta}{\text{grd}}$
22	1.00	30	5.2	6.2	0.1
24	1.00	30	5.0	6.2	0.1
26	1.00	20	4.5	6.2	0.1
28	1.00	20	3.8	6.0	0.1
30	1.00	10	3.2	6.0	0.1
32	1.00	10	2.5	6.0	0.1
34	1.00	10	2.5	6.0	0.1
35	1.00	10	2.5	6.0	0.1

- Additionally record measurement series with the tube high voltages $U = 24$ kV, 26 kV, 28 kV, 30 kV, 32 kV, 34 kV and 35 kV; to save measuring time, use the parameters from table 1 for each series.
- To show the wavelength-dependency, open the "Settings" dialog with the button or F5 and enter the lattice plane spacing for NaCl.
- When you have finished measuring, save the measurement series under an appropriate name by pressing the button or the F2 key.

Measuring example and evaluation

Determining the limit wavelength λ_{\min} as a function of the tube high voltage U :

For each recorded diffraction spectrum (see Fig. 4):

- In the diagram, click the right mouse button to access the evaluation functions of the software "X-ray Apparatus" and select the command "Best-fit Straight Line".
- Mark the curve range to which you want to fit a straight line to determine the limit wavelength λ_{\min} using the left mouse button.
- Save the evaluations under a suitable name using the button or by pressing F2.

Confirming the Duane-Hunt relation and determining Planck's constant

- For further evaluation of the limit wavelengths λ_{\min} determined in this experiment, click on the register "Planck".
- Position the pointer over the diagram, click the right mouse button, fit a straight line through the origin to the curve $\lambda_{\min} = f(1/U)$ and read the slope A from the bottom left corner of the evaluation window (see Fig. 5).

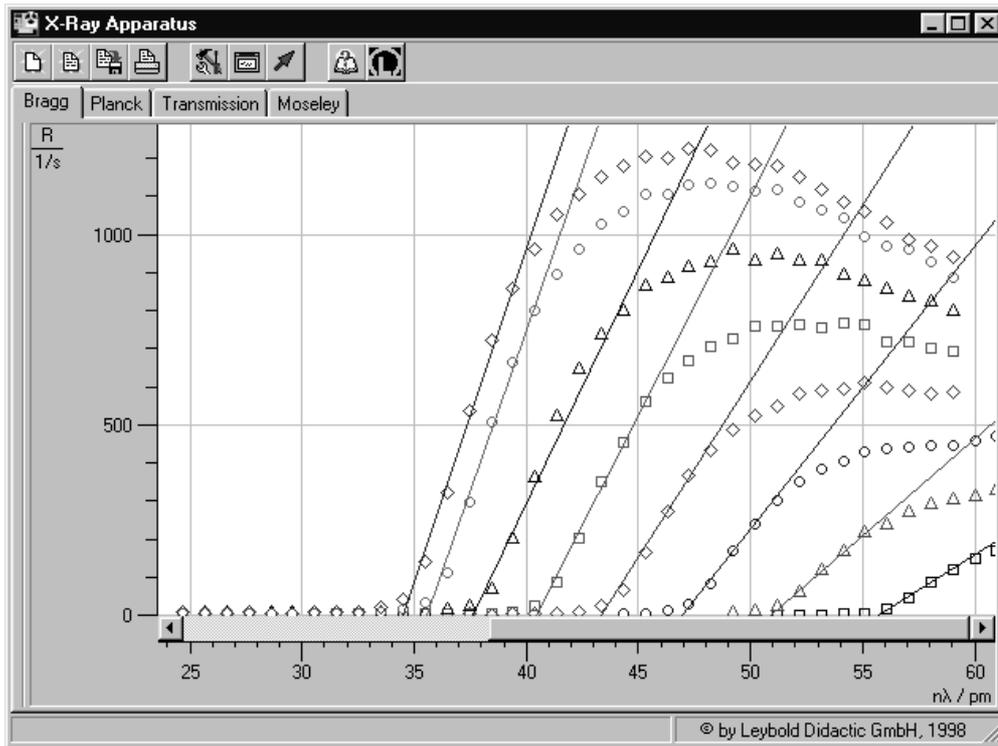


Fig. 4 Sections from the diffraction spectra of x-radiation for the tube high voltages $U = 22, 24, 26, 28, 30, 32, 34$ and 35 kV (from right to left) with best-fit straight line for determining the limit

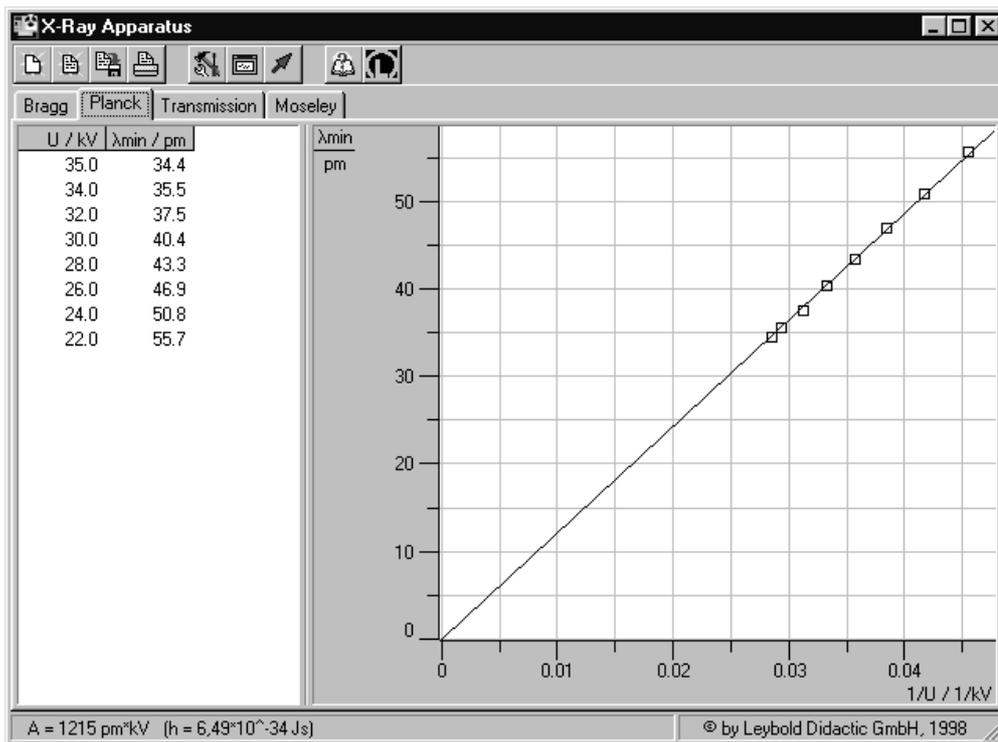


Fig. 5 Evaluation of the data $\lambda_{\min} = f(1/U)$ for confirming the Duane-Hunt relation and determining Planck's constant

The best-fit straight line gives us

$$A = 1215 \text{ pm kV}$$

When we insert this value in equation (VII), we can calculate Planck's constant as:

$$h = 6.49 \cdot 10^{-34} \text{ J s}$$

Literature value:

$$h = 6.626 \cdot 10^{-34} \text{ J s}$$